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CA FOUNDATION MATHEMATICS

INDEX

1 RATIO, PROPORTION, INDICES & LOGARITHM

THEORY

Ratio

- A ratio is a fraction (either proper or improper) which compares two or more quantities of similar kind, which enables us to understand as to how many times one quantity is involved in the other.

- If A : B ($\frac{A}{B}$) is a ratio, then the numerator A is called "Antecedent" and the denominator B is called the "Consequent".

- Ratios must be expressed in the simplest possible form and we can calculate ratios only when the quantities are commensurable (fully quantifiable).

- Two or more ratios can be bridged in order to have a continuous comparison between more than two variables.

- Rule for bridging more than two ratios :

If ,a,b,c,d,e are five Quantities, and

$$\frac{a}{b}=\frac{N_1}{D_1};\frac{b}{c}=\frac{N_2}{D_2};\frac{c}{d}=\frac{N_3}{D_3};\frac{d}{e}=\frac{N_4}{D_4}$$

Then, a:b:c:d:e= $N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$

Let a : b is a ratio, then:

- $\frac{a}{b}>1$ (Ratio of Greater Inequality)
- $\frac{a}{b}<1$ (Ratio of Lesser Inequality)
- $\frac{a}{b}=1$ (Ratio of Equality)

- $a^2 : b^2$ (Duplicate Ratio)
- $a^3 : b^3$ (Triplicate Ratio)
- $\sqrt{a} : \sqrt{b}$ (Sub-Duplicate Ratio)
- $\sqrt[3]{a} : \sqrt[3]{b}$ (Sub-Triplicate Ratio)
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots\ldots$ If then the value of each ratio can be obtained by mean of any one of the following two operations;

a. Each ratio = $\frac{a+c+e+\ldots.}{b+d+f+\ldots.}$ (ADDENDO)

Or

b. Each ratio = $\frac{a-c-e-\ldots.}{b-d-f-\ldots.}$ (SUBTRANDENDO)

INVERSE RATIO:

- IR of a:b is b : a
- IR of a:b:c is bc : ac : ab
- IR of a:b:c:d is bcd : acd : abd : abc

COMPOUND RATIO:

The multiplying effect of all ratios given is known as compound ratio. If a:b and c:d are two ratios, then ac : bd is called the compounded ratio of the two.

Proportion

- Proportion is defined as the equality of two or more ratios. If $\frac{a}{b} = \frac{c}{d}$, in such a case the quantities a,b,c,d are said to be proportional, here 'd' is called the fourth proportional.
- If $\frac{a}{b} = \frac{b}{c}$, then a,b,c are said to be in continued proportion, where 'b' is called the mean proportional and 'c' is called third proportional.
- If $\frac{a}{b} = \frac{b}{c}$ or $b^2 = ac \therefore b = \sqrt{ac}$

IF	THEN	PROPERTY
$\frac{a}{b}=\frac{c}{d}$	ad = bc	PRODUCT OF EXTREMES = PRODUCT OF MEANS
	$\frac{b}{a}=\frac{d}{c}$	INVERTENDO
	$\frac{a}{c}=\frac{b}{d}$	ALTERNENDO
	$\frac{a+b}{b}=\frac{c+d}{d}$	COMPONENDO
	$\frac{a-b}{b}=\frac{c-d}{d}$	DIVIDENDO
	$\frac{a+b}{a-b}=\frac{c+d}{c-d}$	COMPONENDO & DIVIDENDO
	$\frac{a-b}{a+b}=\frac{c-d}{c+d}$	DIVIDENDO & COMPONENDO

CLASSWORK SECTION

1. Two numbers are in the ratio 5 : 6. If 5 is subtracted from each number, the ratio becomes 4 : 5. The numbers are:

   a) 15, 20 b) 5, 10

   c) 10, 15 d) 25, 30

2. Two numbers are in the ratio 3 : 4. If 6 be added to each terms of the ratio, then the new ratio will be 4 : 5. The two numbers are:

   a) 24, 32 b) 18, 24

   c) 15, 20 d) 9, 12

3. Daily earnings of two persons are in the ratio 4 : 5 and their daily expenses are in the ratio 7 : 9. If each saves ₹ 50 per day, their daily incomes are ₹

   a) (40, 50) b) (50, 40)

   c) (400, 500) d) None of these

4. The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are:

   a. 40, 60, 50 b. 50, 45, 55

   c. 55, 35, 60 d. 45, 50, 55

5. Moi earns ₹ 80 in 7 hours and Zen earns ₹ 90 in 12 hours. The ratio of their earnings is:

   a) 32 : 21 b) 23 : 12

   c) 8 : 9 d) None of the above

6. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms in 5 hrs, what is the speed of the first train?

   a) 10 km per hour b) 70 km per hour

   c) 50 km per hour d) None of the above

7. The ratio of the speeds of two trains is 2 : 5. If the distances they travel are in the ratio 5 : 9, find the ratio of the times taken by them.

a. 18 : 25 b. 5 : 4

c. 25 : 18 d. 1:1

8. If x/2 = y/3 = z/7, then find the value of (2x − 5y + 4z) / 2y.

a) 6/23 b) 23/6 c) 3/2 d) 17/6

9. The ratio of the number of 50 paise, Re. 1 and ₹ 5 coins with Mr. Zen is 5 : 2 : 1. If the amount with him is ₹ 38, then the number of Re. 1 coins with him is:

a) 4 b) 8 c) 12 d) 16

10. A boy has five rupee coins, two rupee coins and one rupee coins in the ratio 3:4:5. If he has an amount of ₹ 224, then find the numbers of one rupee coins with the boy.

a) 40 b) 35 c) 20 d) 30

11. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$. Then find the value of each ratio.

a. 1 b. $\frac{1}{2}$ c. $\frac{1}{20}$ d. None of the above

12. A precious stone worth ₹ 15,600 is accidently dropped and broken into three pieces, the weights of which are respectively proportional to 2 : 3 : 5. The value of the stone of this variety varies as the cube of its weight. Calculate the percentage loss thus incurred by this breakage.

a) 80% b) 90%

c) 84% d) 78%

13. An employer reduces the number of employees in the ratio of 19 : 16 and increases their wages in the ratio of 4 : 5. What is the ratio of the wage bill of the employer initially and now?

a. 20 : 19 b. 17 : 16

c. 16 : 17 d. 19 : 20

Types of Ratios:

14. The duplicate ratio of 3 : 4 is

a) $\sqrt{3}:2$ b) 4 : 3

c) 9 : 16 d) None of these

15. The sub-duplicate ratio of 25 : 36 is

a) 6 : 5 b) 36 : 25

c) 50 : 72 d) 5 :6

16. The triplicate ratio of 2 : 3 is

a) 8 : 27 b) 6 : 9

c) 3 : 2 d) None of these

17. The sub-triplicate ratio of 8 : 27 is

a) 27 : 8 b) 24 : 81

c) 2 : 3 d) None of these

18. If p : q is the sub-duplicate ratio of $(p - x^2) : (q - x^2)$, then find the value of x^2.

a) p / (p + q) b) q / (p + q)

c) pq / (p − q) d) pq / p+q

Compound Ratio

19. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is

a) 1 : 1 b) 1 : 5

c) 3 : 8 d) None of these

20. The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is

a) 2 : 7 b) 7 : 2

c) 2 : 21 d) None of these

21. Find the compounded ratio of 275 : 31, inverse of 729 : 1331, duplicate ratio of 2 : 5, triplicate ratio of 9 : 11, sub-duplicate ratio of 961 : 1296, sub-triplicate ratio of 729 : 1331.

a. 1 : 1 b. 1 : 2

c. 275 : 11 d. 31 : 25

Inverse Ratio

22. The Inverse ratio of 11 : 15 is

a) 15:11 b) $\sqrt{11}:\sqrt{15}$

c) 121 : 255 d) None of these

23. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is

a) 5 b) $\sqrt{5}$ c) 7 d) None of these

Joint Ratio

24. If $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$, the ratio a : b : c = ?

a) 4 : 6 : 15 b) 4 : 8 : 15

c) 8 : 12 : 15 d) 8 : 16 : 25

25. If A : B = 2 : 3, B : C = 4 : 5 and C : D = 3 : 7, find A : B : C : D

a) 4 : 6 : 15 : 35 b) 4 : 12 : 15 : 35

c) 8 : 12 : 15 : 35 d) 8 : 16 : 25 : 35

26. If a : b = 3 : 5, b : c = 5 : 4, c : d = 2 : 3 and d is 50% more than e, find the ratio between a and e.

a) 2 : 3 b) 3 : 4

c) 3 : 5 d) 4 : 5

27. Aoi, Boi and Coi work in a company. The ratio of Aoi's age to Boi's age is 11 : 13 and Boi's age to Coi's age is 13:14. If the sum of their ages is 76, what are their respective ages?

a) 33, 39, 42 b) 23, 27, 32

c) 22, 26, 28 d) 24, 28, 30

28. If $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$, then find the value of $\frac{a+b}{b+c}$

a) 8 : 15 b) 20 : 27

c) 3 : 4 d) 27 : 20

29. Ahmedabad, Bombay and Calcutta are three cities. The ratio of average temperature between Ahmedabad and Bombay is 11 : 12 and the average between Ahmedabad and Calcutta is 9 : 8. Then the ratio between the average temperature of Bombay and Calcutta is:

a) 22 : 27 b) 27 : 22

c) 32 : 33 d) None of the above

30. A man distributes his property of ₹ 6,00,000 among his three sons. The share of his first son is thrice that of the second son's share and the share of the second son is twice that of the third son. Find the ratio in which sons share the property.

a) 1 : 2 : 6 b) 3 : 4 : 5

c) 6 : 2 : 1 d) 2 : 4 : 6

Proportion

31. The fourth proportional to 4, 6, 8 is

a) 12 b) 32

c) 48 d) None of these

32. The third proportional to 12, 18 is

a) 24 b) 27

c) 36 d) None of these

33. The mean proportional between 25, 81 is

a) 40 b) 50

c) 45 d) None of these

34. The fourth proportional to 2a, a^3 & c is

a) ac/2 b) ac

c) 2/ac d) $a^2c/2$

35. If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is

a) 6/5 b) 5/6

c) 15/2 d) None of these

36. The mean proportional between $12x^2$ and $27y^2$ is

a) 18xy b) 81xy

c) 8xy d) None of these

37. If x / y = z / w, implies y /x = w / z, then the process is called

a) Dividendo b) Componendo

c) Alternendo d) None of these.

38. If p /q = r / s = p - r / q - s, the process is called

a) Subtrahendo b) Addendo

c) Invertendo d) None of these.

39. If a/b = c/d, then the process (a+b)/(a-b) = (c+d)/(c-d), is called

a) Componendo

b) Dividendo

c) Componendo and Dividendo

d) None of these.

40. If u / v = w / p. then the process (u-v) / (u+v) = (w-p) / (w+p), is called

a) Invertendo

b) Alternendo

c) Addendo

d) None of these.

41. If $\frac{a}{4}=\frac{b}{5}$ then

a) $\frac{a+4}{a-4}=\frac{b-5}{b+5}$ b) $\frac{a+4}{a-4}=\frac{b+5}{b-5}$

c) $\frac{a-4}{a+4}=\frac{b+5}{b-5}$ d) None of these

42. What should be added to each of 3, 15, 38 and 134 so that the number become proportionate to each other.

a) 3 b) 5 c) 7 d) 2

43. In what proportion must rice @ ₹ 3.10/kg be mixed with rice @ ₹ 3.60/kg to make the mixture worth ₹ 3.25/kg?

a. 3 : 5 b. 5 : 3

c. 3 : 7 d. 7 : 3

44. On combining two groups of students having 30 and 40 marks respectively in an exam, the resultant group has an average score of 34. Find the ratio of the number of students in the first group to the number of students in the second group.

a. 2 : 3 b. 3 : 5

c. 5 : 3 d. 3 : 2

45. A merchant has 100 kg of sugar, part of which he sells at 7% profit and the rest at 17% profit. He gains 10% on the whole. Find how much is sold at 7% profit.

a. 30 kg b. 70 kg

c. 55 kg d. 45 kg

PAST YEAR QUESTIONS

46. An alloy is to contain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is

(a) $10\frac{2}{3}$ kg (b) $10\frac{1}{3}$ kg

(c) $9\frac{2}{3}$ kg (d) 9 kg

47. A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coin is double the number of 25 paise coins and four times the numbers of one rupee coins. The numbers of 50 paise coins in the box is

(a) 64 (b) 32 (c) 16 (d) 14

48. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally entire cost of the car, then the share of each of the remaining persons increased by:

(a) 1/9 (b) 1/8 (c) 1/7 (d) 7/8

49. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹ 1,500, then B's income is:

(a) ₹ 6000 (b) ₹ 4500

(c) ₹ 3000 (d) ₹ 7500

50. In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3 : 1. The quantity of water added in the mixture in order to make this ratio 2 : 1 is

(a) 15 litres (b) 10 litres (c) 8 litres (d) 5 litres

51. The third proportional between $(a^2 - b^2)$ and $(a + b)^2$ is :

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$

(c) $\frac{(a-b)^2}{a+b}$ (d) $\frac{(a+b)^3}{a-b}$

52. In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹ 1,60,000 and C gets ₹ 2,50,000. Find the amount received by B?

(a) ₹ 2,00,000 (b) ₹ 2,50,000

(c) ₹ 1,00,000 (d) ₹ 1,50,000

53. The ratio compounded of 4:5 and sub-duplicate of a : 9 is 8 : 15. Then value of a is:

(a) 2 (b) 3 (c) 4 (d) 5

54. Find two numbers such that mean proportional between them is 18 and third proportional between them is 144

(a) 9, 36 (b) 8, 32 (c) 7, 28 (d) 6, 24

55. If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be:

(a) 5 : 8 (b) 8 : 5 (c) 5 : 3 (d) 3 : 5

56. A dealer mixes rice costing ₹ 13.84 per kg. with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3 : 7 (b) 5 : 7 (c) 7 : 9 (d) 9 : 11

57. X, Y, Z together starts a business. If X invests 3 times as much as Y invests and Y invests two third of what Z invests, then the ratio of capitals of X, Y, Z is

(a) 3 : 9 : 2 (b) 6 : 3 : 2

(c) 3 : 6 : 2 (d) 6 : 2 : 3

58. There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3 : 2. Then the number of coins of ₹ 1 is :

(a) 12 (b) 5 (c) 10 (d) 14

59. The ratio of the number of ₹ 5 coins and ₹ 10 coins is 8 : 15. If the value of ₹ 5 coins is ₹ 360, then the number of ₹ 10 coins will be:

(a) 72 (b) 120 (c) 135 (d) 185

HOMEWORK SECTION

1. The inverse ratio of 11 : 15 is
   (a) 15 : 11 (b) $\sqrt{11}:\sqrt{15}$
   (c) 121 : 225 (d) none of these

2. The ratio of two quantities is 3 : 4. If the antecedent is 15, the consequent is
   (a) 16 (b) 60 (c) 22 (d) 20

3. The ratio of the quantities is 5 : 7. If the consequent of its inverse ratio is 5, the antecedent is
   (a) 5 (b) $\sqrt{5}$ (c) 7 (d) none of these

4. The ratio compounded of 2 : 3, 9 : 4, 5 : 6 and 8 : 10 is
   (a) 1 : 1 (b) 1 : 5 (c) 3 : 8 (d) none of these

5. The duplicate ratio of 3 : 4 is
   (a) $\sqrt{3}$: 2 (b) 4 : 3 (c) 9 : 16 (d) none of these

6. The sub-duplicate ratio of 25 : 36 is
   (a) 6 : 5 (b) 36 : 25 (c) 50 : 72 (d) 5 : 6

7. The triplicate ratio of 2 : 3 is
   (a) 8 : 27 (b) 6 : 9 (c) 3 : 2 (d) none of these

8. The sub-triplicate ratio of 8 : 27 is
   (a) 27 : 8 (b) 24 : 81 (c) 2 : 3 (d) none of these

9. The ratio compounded of 4 : 9 and the duplicate ratio of 3 : 4 is
   (a) 1 : 4 (b) 1 : 3 (c) 3 : 1 (d) none of these

10. The ratio compounded of 4 : 9, the duplicate ratio of 3 : 4, the triplicate ratio of 2 : 3 and 9 : 7 is
    (a) 2 : 7 (b) 7 : 2 (c) 2 : 21 (d) none of these

11. The ratio compounded of duplicate ratio of 4 : 5, triplicate ratio of 1 : 3, sub duplicate ratio of 81 : 256 and sub-triplicate ratio of 125 : 512 is

(a) 4 : 512 (b) 3 : 32 (c) 1 : 12 (d) none of these

12. If a : b = 3 : 4, the value of (2a+3b) : (3a+4b) is

(a) 54 : 25 (b) 8 : 25 (c) 17 : 24 (d) 18 : 25

13. Two numbers are in the ratio 2 : 3. If 4 be subtracted from each, they are in the ratio 3 : 5. The numbers are

(a) (16, 24) (b) (4, 6) (c) (2, 3) (d) none of these

14. The angles of a triangle are in ratio 2 : 7 : 11. The angles are

(a) $(20^\circ, 70^\circ, 90^\circ)$

(b) $(30^\circ, 70^\circ, 80^\circ)$

(c) $(18^\circ, 63^\circ, 99^\circ)$

(d) none of these

15. Division of ₹ 324 between X and Y is in the ratio 11 : 7. X & Y would get Rupees

(a) (204, 120) (b) (200, 124)

(c) (180, 144) (d) none of these

16. The ratio of two numbers is 7 : 10 and their difference is 105. The numbers are

(a) (200, 305) (b) (185, 290)

(c) (245, 350) (d) none of these

17. P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is

(a) 22 : 27 (b) 27 : 22 (c) 32 : 33 (d) none of these

18. If x : y = 3 : 4, the value of $x^2y + xy^2 : x^3 + y^3$ is

(a) 13 : 12 (b) 12 : 13 (c) 21 : 31 (d) none of these

19. If p : q is the sub-duplicate ratio of $p-x^2 : q-x^2$ then x^2 is

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$ (c) $\frac{pq}{p+q}$ (d) None of these

20. If 2s : 3t is the duplicate ratio of 2s − p : 3t − p then
(a) $p^2 = 6st$ (b) $p = 6st$ (c) $2p = 3st$ (d) none of these

21. If p : q = 2 : 3 and x : y = 4 : 5, then the value of 5px + 3qy : 10px + 4qy is
(a) 71 : 82 (b) 27 : 28 (c) 17 : 28 (d) none of these

22. The number which when subtracted from each of the terms of the ratio 19 : 31 reducing it to 1 : 4 is
(a) 15 (b) 5 (c) 1 (d) none of these

23. Daily earnings of two persons are in the ratio 4:5 and their daily expenses are in the ratio 7 : 9. If each saves ₹ 50 per day, their daily earnings in ₹ are
(a) (40, 50) (b) (50, 40) (c) (400, 500) (d) none of these

24. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 kms. in 5 hours, the speed of the first train is
(a) 10 Km/hr (b) 50 Km/hr (c) 70 Km/hr (d) none of these

25. The fourth proportional to 4, 6, 8 is
(a) 12 (b) 32 (c) 48 (d) none of these

26. The third proportional to 12, 18 is
(a) 24 (b) 27 (c) 36 (d) none of these

27. The mean proportional between 25, 81 is
(a) 40 (b) 50 (c) 45 (d) none of these

28. The number which has the same ratio to 26 that 6 has to 13 is
(a) 11 (b) 10 (c) 21 (d) none of these

29. The fourth proportional to 2a, a^2, c is
(a) ac/2 (b) ac (c) 2/ac (d) none of these

30. If four numbers 1/2, 1/3, 1/5, 1/x are proportional then x is
(a) 6/5 (b) 5/6 (c) 15/2 (d) none of these

31. The mean proportional between $12x^2$ and $27y^2$ is

(a) 18xy (b) 81xy (c) 8xy (d) none of these

(Hint: Let z be the mean proportional and $z = \sqrt{(12x^2 \times 27y^2)}$

32. If A = B/2 = C/5, then A : B : C is

(a) 3 : 5 : 2 (b) 2 : 5 : 3 (c) 1 : 2 : 5 (d) none of these

33. If a/3 = b/4 = c/7, then a + b + c/c is

(a) 1 (b) 3 (c) 2 (d) none of these

34. If p/q = r/s = 2.5/1.5, the value of ps : qr is

(a) 3/5 (b) 1:1 (c) 5/3 (d) none of these

35. If x : y = z : w = 2.5 : 1.5, the value of (x + z)/(y + w) is

(a) 1 (b) 3/5 (c) 5/3 (d) none of these

36. If (5x − 3y)/(5y − 3x) = 3/4, the value of x : y is

(a) 2 : 9 (b) 7 : 2 (c) 7 : 9 (d) none of these

37. If A : B = 3 : 2 and B : C = 3 : 5, then A : B : C is

(a) 9 : 6 : 10 (b) 6 : 9 : 10 (c) 10 : 9 : 6 (d) none of these

38. If x/2 = y/3 = z/7, then the value of (2x − 5y + 4z)/2y is

(a) 6/23 (b) 23/6 (c) 3/2 (d) 17/6

39. If x : y = 2 : 3, y : z = 4 : 3 then x : y : z is

(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 3 : 2 : 4 (d) none of these

40. Division of ₹ 750 into 3 parts in the ratio 4 : 5 : 6 is

(a) (200, 250, 300) (b) (250, 250, 250)

(c) (350, 250, 150) (d) 8 : 12 : 9

41. The sum of the ages of 3 persons is 150 years. 10 years ago their ages were in the ratio 7 : 8 : 9. Their present ages are

(a) (45, 50, 55) (b) (40, 60, 50)

(c) (35, 45, 70) (d) none of these

42. The numbers 14, 16, 35, 42 are not in proportion. The fourth term for which they will be in proportion is

(a) 45 (b) 40 (c) 32 (d) none of these

43. If x/y = z/w, implies y/x = w/z, then the process is called

(a) Dividendo (b) Componendo
(c) Alternendo (d) none of these

44. If p/q = r/s = p − r/q − s, the process is called

(a) Subtrahendo (b) Addendo
(c) Invertendo (d) none of these

45. If a/b = c/d, implies (a + b)/(a − b) = (c + d)/(c − d), the process is called

(a) Componendo (b) Dividendo
(c) Componendo and Dividendo (d) none of these

46. If u/v = w/p, then (u − v)/(u + v) = (w − p)/(w + p). The process is called

(a) Invertendo (b) Alternendo
(c) Addendo (d) none of these

47. 12, 16, *, 20 are in proportion. Then * is

(a) 25 (b) 14 (c) 15 (d) none of these

48. 4, *, 9, 13½ are in proportion. Then * is

(a) 6 (b) 8 (c) 9 (d) none of these

49. The mean proportional between 1.4 gms and 5.6 gms is

(a) 28 gms (b) 2.8 gms (c) 3.2 gms (d) none of these

50. If $\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$ then $\frac{a+b+c+}{c}$ is

(a) 4 (b) 2 (c) 7 (d) none of these.

51. Two numbers are in the ratio 3 : 4; if 6 be added to each terms of the ratio, then the new ratio will be 4 : 5, then the numbers are

(a) 14, 20 (b) 17, 19 (c) 18 and 24 (d) none of these

52. If $\frac{a}{4} = \frac{b}{5}$ then

(a) $\frac{a+4}{a-4} = \frac{b-5}{b+5}$ (b) $\frac{a+4}{a-4} = \frac{b+5}{b-5}$

(c) $\frac{a-4}{a+4} = \frac{b+5}{b-5}$ (d) none of these

53. If a : b = 4 : 1 then + is $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}$ is

(a) 5/2 (b) 4 (c) 5 (d) none of these

54. If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ then

(b - c)x + (c - a)y + (a - b)z is

(a) 1 (b) 0 (c) 5 (d) none of these

HOMEWORK SOLUTION

1. (a) 15 : 11

2. (d) 20

Ratio = $\frac{3}{4}$, antecedent = 3 × 5 = 15

∴ consequent = 4 × 5 = 20

3. (c) 7

4. (a) 1 : 1

5. (c) 9 : 16

6. (d) 5 : 6

7. (a) 8 : 27

8. (c) 2 : 3

9. (a) 1/4

Compound ratio = $\frac{4}{9} \times \frac{9}{16} = \frac{1}{4}$

10. (c) 2 : 27

Compound ratio = $\frac{4}{9} \times \frac{9}{16} \times \frac{8}{27} \times \frac{9}{7} = \frac{2}{21}$

11. (d) None of these

Compound ratio = $\frac{16}{25} \times \frac{1}{27} \times \frac{9}{16} \times \frac{5}{8} = \frac{1}{120}$

12. (d) 18 : 25

Here, a : b = 3 : 4

∴ a = 3, b = 4

∴ Value of 2a + 3b : 3a + 4b

= 2(3) + 3(4) : 3(3) + 4(4)

= 18 : 25

13. (a) 16, 24

Let numbers are 2 : 3 = 2x : 3x

$\square$ If 4 subtract from each

$\square$ $\frac{2x-4}{3x-4} = \frac{3}{5}$

$\square$ $5(2x-4) = 3(3x-4)$

$\square$ $10x - 20 = 9x - 12$

$\square$ $x = 8$

$\square$ the numbers are = 2x, 3x

= 16, 24

14. (c) (18°, 63°, 99°)

Angles of triangle = 2 : 7 : 11

= 2x, 7x, 11x

Let 2x + 7x + 11x = 180

$\square$ 20x = 180

$\square$ x = 9

$\square$ Angles of triangle = 2x, 7x, 11x

= 18, 63, 99

15. (d) None of these Trial and error

16. (c) (245, 350) Trial and error

17. (b) 27 : 22

Here, P : Q = 11 : 12, P : R = 9 : 8

$\square$ Q : P = 12 : 11

Joint ratio = Q : P	P : R
9 × (12 : 11)	(9 : 8) × 11
= 108 : 99	99 : 88

$\square$ Ratio of Q : R = 108 : 88

= 27 : 22

18. (b) 12 : 13

Here, x : y = 3 : 4

$\square$ x = 3, y = 4

the value of $x^2y + xy^2 : x^3 + y^3$

$= (3)^2(4) + 3(4)^2 : (3)^3 + (4)^3$

$= 36 + 48 : 27 + 64$

$= 84 : 91$

$= 12 : 13$

19. (c) pq / p + q

$$\frac{p}{q} = \frac{\sqrt{p - x^2}}{\sqrt{q - x^2}}$$

$$\therefore \frac{p^2}{q^2} = \frac{p - x^2}{q - x^2}$$

$\square$ $p^2(q - x^2) = q^2(p - x^2)$

$\square$ $p^2q - p^2x^2 = pq^2 - q^2 . x^2$

$\square$ $p^2q - pq^2 = p^2x^2 - q^2x^2$

$\square$ $pq(p - q) = x^2(pp - qq)$

$\square$ $pq(p - q) = x^2(p - q)(p + q)$

$\square$ $x^2 = \frac{pq}{p + q}$

20. (a) $p^2 = 6$ st

$$\frac{2s}{3t} = \frac{(2s - p)^2}{(3t - p)^2}$$

$$\therefore \frac{2s}{3t} = \frac{4s^2 - 4sp + p^2}{9t^2 - 6tp + p^2}$$

$\square$ $2s(9t^2 - 6tp + p^2) = 3t(4s^2 - 4sp + p^2)$

$\square$ $18t^2s - 12ps + 2p^2s = 12ts^2 - 12tps + 3pt$

$\square$ $18t^2s - 12 + s^2 = 3p^2t - 2p^2s$

$\square$ $6ts(3t - 2s) = p^2(3t - 2s)$

$\square$ $p^2 = 6st$

21. (c) 17 : 28

$p : q = 2 : 3, x : y = 4 : 5$

$\therefore$ the value of $5px + 3qy : 10px + 4qy$

$= 5(2)(4) + 3(3)(5) : 10(2)(4) + 4(3)(5)$

$= 85 : 140 = 17 : 28$

22. (a) 15 Trial and error

$$\frac{19-15}{31-15} = \frac{4}{16} = \frac{1}{4}$$

23. (c) (400, 500) Trial and error

24. (c) 70 km/hrs

Speed d 2nd train = $\frac{\text{Distance}}{\text{Time}}$

$= \frac{000}{5}$

$\therefore$ Speed of 2nd train = 80 km/hr

Speed ratio = $\frac{S_1}{S_2} = \frac{7}{8}$

$\therefore S_1 = \frac{7}{8} \times S_2$

$= \frac{7}{8} \times 80$

$\therefore S_1 = 70$ km/hr.

25. (a) 12

$\frac{4}{6} = \frac{8}{x}$

$\therefore x = \frac{8 \times 6}{4}$

$\therefore x = 12$

26. (b) 27

$\frac{12}{18} = \frac{18}{x}$

$\therefore x = \frac{18 \times 18}{12}$

$\square x = 27$

27. (c) 45

Mean proportion = $\sqrt{25 \times 81}$

$= 45$

28. (d) None of these

$\frac{x}{26} = \frac{6}{13}$

$\therefore x = \frac{6 \times 26}{13}$

$\square x = 12$

29. (a) $\frac{ac}{2}$

$\frac{2a}{a^2} = \frac{c}{x}$

$\therefore x = \frac{ca^2}{2a}$

$x = \frac{ac}{2}$

30. (c) $\frac{15}{2}$

$\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{\frac{1}{5}}{\frac{1}{x}}$

$\therefore \frac{3}{2} = \frac{x}{5}$

$\therefore x = \frac{3 \times 5}{2} = \frac{15}{2}$

31. (a) 18xy

Mean proportion = $\sqrt{12x^2 + 27y^2}$

$= \sqrt{324 . x^2 . y^2}$

$= 18xy$

32. (c) 1 : 2 : 5

Here A = $= \frac{B}{2} = \frac{C}{5}$

A = 1, B = 2, C = 5

$\square$ A : B : C = 1 : 2 : 5

33. (c) 2

Here, $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$

$\square$ a = 3, b = 4, c = 7

$\frac{a + b + c}{c} = \frac{3 + 4 + 7}{7} = \frac{14}{7} = 2$

34. (b) 1 : 1

$\frac{p}{q} = \frac{r}{s} = \frac{2.5}{1.5}$

$\square$ p = r = 2.5, q = s = 1.5

$\frac{ps}{qr} = \frac{(2.5)(1.5)}{(1.5)(2.5)} = \frac{1}{2}$

35. (c) $\frac{5}{3}$

$\frac{x}{y} = \frac{z}{2} = \frac{2.5}{1.5}$

x = z = 2.5, y = w = 1.5

the value of $\frac{x + z}{y + w} = \frac{2.5 + 2.5}{1.5 + 1.5} = \frac{5}{3}$

36. (d) $\frac{27}{29}$

Here, $\frac{5x - 3y}{5y - 3x} = \frac{3}{4}$

- $4(5x - 3y) = 3(5y - 3x)$
- $20x - 12y = 15y - 9x$
- $29x = 27y$

- $\frac{x}{y} = \frac{27}{29}$

37. (a) 9 : 6 : 10

A : B B : C

3 × (3 : 2) (3 : 5) × 2

- 9 : 6 6 : 10
- A : B : C = 9 : 6 : 10

38. (d) $\frac{17}{16}$

$\frac{x}{2} = \frac{y}{3} = \frac{z}{7}$

- x = 2, y = 3, z = 7
- The value of $\frac{2x - 5y + 4z}{2y}$

$= \frac{2(2) - 5(3) + 4(7)}{2(3)}$

$= \frac{17}{6}$

39. (d) 8 : 12 : 9

x : y y : z

4 × (2 : 3) (4 : 3) × 3

- 8 : 12 12 : 9
- x : y : z = 8 : 12 : 9

40. (a) (200, 250, 300) Trial and error

41. (a) (45, 50, 55) Trial and error

42. (b) 40

$\frac{14}{16} = \frac{35}{x}$

$\therefore x = \frac{35 \times 16}{14}$

$x = 40$

43. (d) None of these

44. (a) Subtrahendo

45. (c) Componendo & Dividendo

46. (d) None of these

47. (c) 15

$\frac{12}{16} = \frac{x}{20}$

$\therefore x = \frac{12 \times 20}{16}$

$x = 15$

48. (a) 6

$\frac{4}{x} = \frac{9}{13.5}$

$\therefore x = \frac{4 \times 13.5}{9}$

$x = 6$

49. (b) 2.8

Mean proportion = $\sqrt{1.4 \times 5.6}$

$= 2.8$

50. (b) 2

$\frac{a}{4} = \frac{b}{5} = \frac{c}{9}$

$\square$ a = 4, b = 5, c = 9

$\therefore \frac{a+b+c}{c} = \frac{4+5+9}{9} = 2$

51. (c) 18, 24

$\frac{3x + 6}{4x + 6} = \frac{4}{5}$

- $5(3x + 6) = 4(4x + 6)$
- $15x + 30 = 16x + 24$
- $6 = x$
- numbers = $3x = 3(6) = 18$

$4x = 4(6) = 24$

52. (b) $\frac{a + 4}{a - 4} = \frac{b + 5}{b - 5}$

53. (a) $\frac{5}{2}$

Here, $\frac{a}{b} = \frac{4}{1}$

$\therefore \frac{\sqrt{a}}{\sqrt{b}} = \frac{2}{1}, \frac{\sqrt{b}}{\sqrt{a}} = \frac{1}{2}$

$\therefore$ Value of $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \frac{2}{1} + \frac{1}{2}$

$= \frac{4 + 1}{2}$

$= \frac{5}{2}$

54. (b) 0

Cyclical terms in base.

Indices, Surds and Logarithms

THEORY

$a^x = N$

a = base

x = Power/Exponent/Index

N = Product

[But, $a \neq 0, 1, \pm\infty$]

Theory of Indices deals with the various changes in power, during various mathematical operations.

Basic Rules

1. $a^m \times a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$; m is added n times

4. $(ab)^m = a^m x b^m$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

6. $a^0 = 1$

7. $a^{-n} = \frac{1}{a^n}$

8. If $a^m = a^n \Rightarrow m = n$; where, $a \neq 0, 1, -1, \pm\infty$

9. For $a^m = b^m$ if m ≠ 0 then

(i) $a = b$ (when m is odd)

(ii) $a = \pm b$ (when m is even)

10. $a^x = N$

$\Rightarrow a = N^{\frac{1}{x}} = \sqrt[x]{N}$

11. $(i) 0^a = 0$

$(ii) 1^a = 1$

$(iii) a^1 = a$

$(iv) a^0 = 1$

$(v) 0^0$ *has* no meaning

Basic Formulae

1. $(a+b)^2 = a^2 + 2ab + b^2$

2. $(a-b)^2 = a^2 - 2ab + b^2$

3. $a^2 - b^2 = (a+b)(a-b)$

4. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

5. $(a+b)^2 - (a-b)^2 = 4ab$

6. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

7. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

8. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

11. If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

12. If $a^3+b^3+c^3=3abc, then$ *either* $a+b+c=0$ or a = b = c
but both the results cannot hold true simultaneously

Rational Numbers, Irrational Numbers & Surds

- A Rational Number is a number which can be expressed in the form p/q, where q ≠ 0; p & q are integers and p and q are prime to each other, i.e., there is no common factor between p & q, other than 1.

- Any terminating and recurring decimals are rational numbers.

- Thus any non-recurring and non-terminating decimals are irrational numbers, and when the irrational numbers are expressed in radical form (root form), it is known as "Surds".

- Thus all the surds are irrational, but all irrational numbers are not surds.

- The numbers whose perfect root can be evaluated are rational quantities and numbers for which perfect roots cannot be evaluated are irrational quantities.

Order of Surds

If $\sqrt[k]{m}=(m)^{\frac{1}{k}}$ is a surd, then, it is said to be a surd of order "k".

Pure Surds and Mixed Surds

In case of pure surds, entire expression is kept within the radical sign. In mixed surds, it is expressed as a product of one rational and one irrational quantity.

Example:
$\sqrt{7}$ is a pure surd; $\sqrt{12}=\sqrt{4x3}=2\sqrt{3}$ is a mixed surd.

Conjugate of a Surd

If $\left(a+\sqrt{b}\right)$ or $\left(\sqrt{a}+\sqrt{b}\right)$ are surds, their respective conjugates would be given by,

$\left(a-\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$ and vice-versa.

Rationalization of Surds

Rationalization is a process, where we convert the irrational part of the surd into a rational quantity, with help of its conjugate.

Note: 1

- Rational + Rational = Rational
- Rational – Rational = Rational
- Rational x Rational = Rational
- Rational ÷ Rational = Rational

Note: 2

- Irrational + Irrational = Irrational
- Irrational – Irrational = Rational (only when the quantities are equal); otherwise –
- Irrational – Irrational = Irrational
- Irrational x Irrational = May be Rational or Irrational
- Irrational ÷ Irrational = May be Rational or Irrational

Note: 3

- Rational + Irrational = Irrational
- Rational – Irrational = Irrational
- Rational x Irrational = Irrational
- Rational ÷ Irrational = Irrational

Square Root of Surds

- The square root of a surd is always a surd.

- Every answer for square root must contain +ve or –ve sign and in the absence of +/- sign, "none of these" will be marked as answer.

- If the given surd, whose square root is to be evaluated is in the form $\left(a \pm \sqrt{b}\right)$, then the answer will also be in the form $\pm\left(x \pm \sqrt{y}\right)$.

- Square the options, in order to get the question back.

INDICES

1. The value of $4/(32)^{1/5}$ is

   (a) 8 (b) 2

   (c) 4 (d) none of these

2. $2^{1/2} \cdot 4^{3/4}$ is equal to

   (a) a fraction (b) a positive integer

   (c) a negative integer (d) none of these

3. The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is

   (a) y^{a+b} (b) y

   (c) 1 (d) $1/y^{a+b}$

4. The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is

   (a) 9/4 (b) 4/9

   (c) 2/3 (d) none of these

5. $[\{(2)^{1/2} \cdot (4)^{3/4} \cdot (8)^{5/6} \cdot (16)^{7/8} \cdot (32)^{9/10}\}^4]^{3/25}$ is

   (a) A fraction (b) an integer

   (c) 1 (d) none of these

6. $[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-1/2}$ is equal to

   (a) x (b) 1/x

   (c) 1 (d) none of these

7. If $a^3 - b^3 = (a-b)(a^2+ab+b^2)$, then the simplified form of

$$\left[\frac{x^l}{x^m}\right]^{l^2+lm+m^2} \times \left[\frac{x^m}{x^n}\right]^{m^2+mn+n^2} \times \left[\frac{x^n}{x^l}\right]^{l^2+ln+n^2}$$

   (a) 0 (b) 1 (c) x (d) none of these

8. The value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

   (a) 1 (b) 0 (c) 2 (d) none of these

9. The value of $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

(a) 1 (b) 0 (c) −1 (d) none of these

10. Using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ tick the correct of these when $x = p^{1/3} - p^{-1/3}$

(a) $x^3 + 3x = p + 1/p$

(b) $x^3 + 3x = p - 1/p$

(c) $x^3 + 3x = p + 1$ (d) none of these

11. If $x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$, then $3x^3 - 9x$ is

(a) 15 (b) 10

(c) 12 (d) none of these

12. If $a^x = b$, $b^y = c$, $c^z = a$, then xyz is

(a) 1 (b) 2

(c) 3 (d) none of these

13. If $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1, then the value of p + q + r is

(a) 1 (b) 0

(c) 1/2 (d) none of these

14. On simplification, $1/(1 + a^{m-n} + a^{m-p}) + 1/(1 + a^{n-m} + a^{n-p}) + 1/(1 + a^{p-m} + a^{p-n})$ is equal to

(a) 0 (b) a (c) 1 (d) 1/a

15. If $2^x = 3^y = 6^{-z}$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

(a) 1 (b) 0 (c) 2 (d) none of these

16. On simplification $\frac{2^{x+3} \times 3^{2x-y} \times 5^{x+y+3} \times 6^{y+1}}{6^{x+1} \times 10^{y+3} \times 15^x}$ reduces to

(a) −1 (b) 0 (c) 1 (d) 10

17. If $\frac{9^y.3^2.(3^{-y})^{-1} - 27^y}{3^{3x}.2^3} = \frac{1}{27}$ then x − y is given by

(a) −1 (b) 1 (c) 0 (d) none

18. Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is given by

(a) 0 (b) −1 (c) 3 (d) 1

19. Show that reduces to $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$

(a) 1 (b) 3 (c) 0 (d) 2

20. The value of z is given by the following if $z^{z\sqrt{z}} = (z\sqrt{z})^z$

(a) 2 (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) $\frac{9}{4}$

21. If $(5.678)^x = (0.5678)^y = 10^z$ then

(a) $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 1$ (b) $\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = 0$

(c) $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = -1$ (d) None

22. If $3^a = 5^b = (75)^c$, then the value of ab − c(2a + b) reduces to

(a) 1 (b) 0 (c) 3 (d) 5

23. If $2^a = 4^b = 8^c$ and abc = 288 then the value $\frac{1}{2a} + \frac{1}{4b} + \frac{1}{8c}$ is given by

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{11}{96}$ (d) $-\frac{11}{96}$

24. If $ax^{2/3} + bx^{1/3} + c = 0$ then the value of $a^3x^2 + b^3x + c^3$ is given by

(a) 3abcx (b) −3abcx (c) 3abc (d) −3abc

25. $x^{a^2b^{-1}c^{-1}}.x^{b^2c^{-1}a^{-1}}.x^{c^2a^{-1}b^{-1}} - x^3$ would reduce to zero a + b + c is given by

(a) 1 (b) −1 (c) 0 (d) None

26. If $a^b = b^a$, then the value of $\left(\frac{a}{b}\right)^{\frac{a}{b}} - a^{\frac{a}{b}-1}$ reduces to

(a) a (b) b (c) 0 (d) None

SURDS

27. If $a = 3 + 2\sqrt{2}$ then the value of $a^{1/2} + a^{-1/2}$ is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) $2\sqrt{2}$ (d) $-2\sqrt{2}$

28. If $a = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$ then the value of $[a(a-14)]^2$ is

(a) 14 (b) 7 (c) 2 (d) 1

29. The square root of $3 + \sqrt{5}$ is

(a) $\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}$ (b) $-\left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}\right)$

(c) Both the above (d) None

30. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then the value of $a^2 + b^2$ is

(a) 10 (b) 100 (c) 98 (d) 99

PAST YEAR QUESTIONS

31. Value of $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$ is:

(a) $a + \frac{1}{a}$ (b) $a - \frac{1}{a}$ (c) $a^2 + \frac{1}{a^2}$ (d) $a^2 - \frac{1}{a^2}$

32. If $2^x - 2^{x-1} = 4$ then x^x is equal to:

(a) 7 (b) 3 (c) 27 (d) 9

33. $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) 1/2 (b) 3/2 (c) 2/3 (d) 1/3

34. If $2^x \times 3^y \times 5^z = 360$. Then what is the value of x, y, z ?

(a) 3, 2, 1 (b) 1, 2, 3 (c) 2, 3, 1 (d) 1, 3, 2

35. If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$

(a) abc (b) 9abc

(c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$

36. If $(25)^{150} = (25x)^{50}$, then the value of x will be:

(a) 5^3 (b) 5^4 (c) 5^2 (d) 5

37. If $p^x = q$, $q^y = r$ and $r^z = p^6$, then the value of xyz will be :

(a) 0 (b) 1 (c) 3 (d) 6

SPECIAL TYPE OF QUESTIONS

1. (i) The value of $\sqrt{20+\sqrt{20+\sqrt{20+}.....to\infty}}$ is

(a) 5 (b) 4 (c) 20 (d) None

(ii) The value of $\sqrt{20-\sqrt{20-\sqrt{20-}.....to\infty}}$ is

(a) 5 (b) 4 (c) 20 (d) None

(iii) $\sqrt{20\sqrt{20\sqrt{20.....}}}\ to\infty$

(a) 5 (b) 4 (c) 20 (d) None

(iv) The value of $\sqrt{8\div\sqrt{8\div\sqrt{8\div.....}}}\ to\infty$

(a) 2 (b) 8 (c) 6 (d) None

(v) The value of $\sqrt{8\sqrt{8\sqrt{8.....}}}$

(a) 4 (b) 2 (c) 8 (d) None

2. If xyz = 1 then the value of $\frac{1}{1+x+y^{-1}}+\frac{1}{1+y+z^{-1}}+\frac{1}{1+z+x^{-1}}$ is

(a) 1 (b) 0 (c) −2 (d) None

3. If x = $\sqrt{2-\sqrt{2-\sqrt{2-}.....\infty}}$ the value of X is given by

(a) − 2 (b) 1 (c) 2 (d) 0

4. If $x=\sqrt{7\sqrt{7\sqrt{7.....\infty}}}$ the value of X is given by

(a) − 3 (b) 3 (c) 12 (d)

5. Simplify $\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}}$ for a = $3^{16/15}$

(a) 0; (b) 2; (c) 3; (d) None

Logarithms

THEORY

If $a^x=N$, then $x=\log_a N$; * $a \neq 0,1, \pm\alpha$ and for the purpose of log, any negative quantity.
* x is called the logarithm of N (product) to the base "a".

Base "a"

- The base "a" of log can be any positive real number except 1.
- The base of log can be clearly divided into two parts:
  - $0 < a < 1$ (the proper fraction)
  - $a > 1$ (positive integer / mixed fraction)
- Unless otherwise specified, the base of log is always taken to be 10 and this is known as Common Logarithm.
- For theoretical purpose, the base is always taken to be "e", where "e" is a constant and this is known as "Natural Logarithm".
- Common Logarithms are used for numerical calculations and Natural Logarithms are used in calculus.

Basic Rules

1. $\log_a mn = \log_a m + \log_a n$

2. $\log_a \frac{m}{n} = \log_a m - \log_a n$

3. $\log_a m^n = n \log_a m$

4. $\log_a a = 1$

5. $\log_a 1 = 0$

6. $\log_a 0 =$ Undefined

7. $\log_a -ve =$ Undefined

8. $\log_a m = \log_a n \Rightarrow m = n$

Change of Base in Logarithms

1. $\log_b a = \frac{\log_m a}{\log_m b}$ (m can be any common base) $(m \neq 0, 1, \pm\alpha$,-ve value)

2. $\log_a b = \frac{1}{\log_b a}$

3. $a^{\log_a x} = x$

Nature of Log Values

- All the values which are obtained from log tables are irrational numbers provided the numbers are not 10 or in the form of 10^n.
- $\log_b a$ is a rational quantity only when, $\frac{\log a}{\log b}$ is rational.
- If K is a number, then its log value, logK can be divided into two parts: a) Integral Part, b) Fractional Part.
- The integral part is called "Characteristics" and the fractional part is called "Mantissa".
- The integral characteristics part can be positive or negative or zero but not a fraction.
- The values of mantissa are always positive fractions.
- The values for mantissa are obtained from log tables.
- Characteristics are to be calculated before we evaluate mantissa from the log table.
- Value of characteristics = number of significant digits before decimal – 1

CLASSWORK SECTION

1. $\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000$ is
   a) 15
   b) $\log_{10} 11111$
   c) $\log_{10} 1111$
   d) $14\log_{10} 100$

2. If $\log\left(\frac{a}{b}\right)+\log\left(\frac{b}{a}\right)=\log(a+b)$, then which of the following is true?
   a) a + b = 1
   b) a + b = 0
   c) a = b
   d) a − b = 1

3. Find the value of $\log_{10}\left(\frac{4}{25}\right)+\log_{10}\left(\frac{125}{7}\right)-\log_{10}\left(\frac{2}{7}\right)$.
   a) 1 b) 4
   c) 41 d) None of the above

4. $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$ equals
   a) 18
   b) 1
   c) 3
   d) None of the above

5. $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}$ =
   a) log 2
   b) log 3
   c) log 5
   d) None of the above

6. If $\log_{10}[98 + \sqrt{x^2-12x+36}] = 2$, then x =
   a) 4
   b) 8
   c) 12
   d) 4, 8

7. If $\log_5\left(x^2+x\right)-\log_5(x+1)=2$; then find the value of x.

a) 5

b) 1/5

c) 5^2

d) None of the above

8. If $\left(\frac{21}{10}\right)^x$ = 2, then x = ?

a) $\frac{\log 2}{\log 3+\log 7+1}$

b) $\frac{\log 2}{\log 3+\log 7-1}$

c) $\frac{\log 2}{\log 7+\log 3+2}$

d) None of the above

9. Evaluate: $x^{\log y-\log z}.\ y^{\log z-\log x}.\ z^{\log x-\log y}$.

a. 0 b. 1 c. 2 d. − 1

10. The value of is $a^{\log b/c}.\ b^{\log c/a}.\ c^{\log a/b}$

a) 0 b) 1 c) -1 d) None

11. Given log2 = 0.3010 and log3 = 0.4771, find the value of log6.

a) 0.9030 b) 0.9542

c) 0.7781 d) None of the above

12. Given that $\log_{10}2 = x$ and $\log_{10}3 = y$, the value of $\log_{10}60$ is expressed as:

a) x + y + 1 b) x − y + 1

c) x − y − 1 d) None of the above

13. Given logx = m + n and logy = m − n, the value of log $(10x/y^2)$ is expressed in terms of m and n as:

a) 1- m + 3n b) m − 1 + 3n

c) m + 3n + 1 d) None of the above

14. If $\log\left(\frac{x+y}{5}\right)=\frac{1}{2}(\log x+\log y)$, then $\frac{x}{y}+\frac{y}{x}$ =

a) 20 b) 23 c) 22 d) 21

15. If $\log a=\frac{1}{2}\log b=\frac{1}{5}\log c$, then find the value of $a^4b^3c^{-2}$.

a) 0 b) − 1 c) 1 d) None of the above

16. Find the value of $\log_{2\sqrt{3}} 1728$.

a) 2 b) 6 c) 1 d) None of the above

17. On solving the equation logt + log (t - 3) = 1 we get the value of t as

a) 5 b) 2 c) 3 d) 0

18. For any three consecutive integers x, y, z. the equation log (1 + xz) – 2logy = 0 is:

a) True b) False

c) Sometimes true d) Cannot be determined in case of cyclic order

19. If $\log_2 (\log_3 (\log_2 x)) = 1$, then x =

a) 512 b) 128 c) 12 d) 0

20. If $\log_{0.5} (\log_x (\log_4 32)) = 2$, then x =

a) 5/2 b) 625/16 c) 25/4 d) None of the above

21. If $x = \log_a bc$; $y = \log_b ca$; $z = \log_c ab$, then the value of xyz – x – y – z is:

a) 1 b) 2 c) – 1 d) 0

22. Find the value of $\log_5 5.\log_4 9.\log_3 2$.

a) 1 b) 2 c) 5 d) None of the above

23. Find the value of $(\log_b a \times \log_c b \times \log_a c)^3$

a). 1 b) 2 c) 3 d) None of the above

24. If $\log_4 x + \log_2 x = 6$, then the value of x is

a) 2 b) 4 c) 8 d) 16

25. If $\log_{10\sqrt{x}} = 2 \log_x 10$, then a possible value of x is given by:

a) 10 b) $\frac{1}{100}$ c) $\frac{1}{1000}$ d) None of the above

26. Evaluate : $a^{\frac{1}{\log_b a}}$

a) a b) b c) a + b d) None of the above

27. Find the value of the following expression: $a^{\log_a b.\log_b c.\log_c d.\log_d t}$

a) t b) abcdt c) a+b+c+d+t d) None of the above

28. The value of $\left[\frac{1}{\log_{\frac{p}{q}} x}+\frac{1}{\log_{\frac{q}{r}} x}+\frac{1}{\log_{\frac{r}{p}} x}\right]$ is ...?

a) 3 b) 2 c) 1 d) None of the above

29. $\log_2 \log_{\sqrt{2}} \log_3 81 = ?$

a) 3 b) 2 c) 1 d) 0

30. If MOI = $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$, then MOI equals:

a) 3 b) 5 c) 7 d) 25

31. Given log2 = 0.30103, the number of digits in 2^{50} is

a) 14 b) 16 c) 18 d) 25

32. $\log_2 5$

a) An integer b) A rational number

c) An irrational number d) A prime number

33. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$

a) log 2 b) 1 c) 0 d) None of the above

34. The value of $\log_2 [\log_2 [\log_3 (\log_3 27^3)]]$ is equal to

(a) 1 (b) 2 (c) 0 (d) none of these

35. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, these x is equal to

(a) 8 (b) 4 (c) 16 (d) none of these

36. Given that $\log_{10} 2 = x$, $\log_{10} 3 = y$, then $\log_{10} 1.2$ is expressed in terms of x and y as

(a) x + 2y − 1 (b) x + y − 1 (c) 2x + y − 1 (d) none of these

37. The value of $\log_8 25$ given log 2 = 0.3010 is

(a) 1 (b) 2 (c) 1.5482 (d) none of these

38. If $a = b^2 = c^3 = d^4$ then the value of $\log_a (abcd)$ is

(a) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ (b) $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}$

(c) 1 + 2 + 3 + 4 (d) None

39. The sum of the series $\log_a b + \log_{a^2} b^2 + \log_{a^3} b^3 + \ldots + \log_{a^n} b^n$ is given by

(a) $\log_a b^n$ (b) $\log_{a^n} b$ (c) $\log_{a^n} b^n$ (d) None

40. If $a^2 + b^2 = 7ab$ then the value of is $\log \frac{a+b}{3} - \frac{\log a}{2} - \frac{\log b}{2}$

(a) 0 (b) 1 (c) −1 (d) 7

41. If $a^3 + b^3 = 0$ then the value of $\log(a + b) - \frac{1}{2}(\log a + \log b + \log 3)$ is equal to

(a) 0 (b) 1 (c) −1 (d) 3

PAST YEAR QUESTIONS

42. The value of $2 \log x + 2 \log x^2 + 2 \log x^3 + \ldots + 2 \log x^n$ will be:

(a) $\frac{n(n+1)\log x}{2}$ (b) $n(n + 1) \log x$

(c) $n^2 \log x$ (d) none of these

43. If n = m! where ('m' is a positive integer ≥ 2) then the value of :

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\ldots.+\frac{1}{\log_m n}$$

(a) 1 (b) 0 (c) −1 (d) 2

44. Which of the following is true. If $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$

(a) $\log(ab + bc + ca) = abc$

(b) $\log\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc$

(c) $\log (abc) = 0$

(d) $\log (a + b + c) = 0$

45. For what value of x, the equation $(\log_{\sqrt{x}} 2)^2 = \log_x 2$ is true?

(a) 16 (b) 32 (c) 8 (d) 4

46. If $x = \log_{24} 12$, $y = \log_{36} 24$ and $z = \log_{48} 36$, then $xyz + 1 =$ _____

(a) 2xy (b) 2xz (c) 2yz (d) 2

47. The value of $\log (1^3 + 2^3 + 3^3 + \ldots n^3)$ is equal to:

(a) $3 \log 1 + 3 \log 2 + \ldots + 3 \log n$

(b) $2 \log n + 2 \log (n + 1) - 2 \log 2$

(c) $\log n + \log (n + 1) + \log (2n + 1) - \log 6$

(d) 1

SPECIAL TYPE OF QUESTIONS

1. Find the simplest value of $\log_{17}\sqrt{17\sqrt{17\sqrt{17.....\infty}}}$

   (a) 1; (b) – 1; (c) 0; (d) None

2. If $\log_{1000} x = \frac{-1}{4}$, then x is given by:

   (a) 1/100 (b) 1/10 (c) 1/20 (d) None of these

HOMEWORK SECTION
(INDICES & LOG)

1. $4x^{-1/4}$ is expressed as
   (a) $-4x^{1/4}$ (b) x^{-1} (c) $4/x^{1/4}$ (d) none of these

2. The value of $8^{1/3}$ is
   (a) $3\sqrt{2}$ (b) 4 (c) 2 (d) none of these

3. The value of $2 \times (32)^{1/5}$ is
   (a) 2 (b) 10 (c) 4 (d) none of these

4. The value of $4/(32)^{1/5}$ is
   (a) 8 (b) 2 (c) 4 (d) none of these

5. The value of $(8/27)^{1/3}$ is
   (a) 2/3 (b) 3/2 (c) 2/9 (d) none of these

6. The value of $2(256)^{-1/8}$ is
   (a) 1 (b) 2 (c) 1/2 (d) none of these

7. $2^{½}.\ 4^{¾}$ is equal to
   (a) a fraction (b) a positive integer
   (c) a negative integer (d) none of these

8. $\left(\frac{81x^4}{y^{-8}}\right)^{\frac{1}{4}}$ has simplified value equal to
   (a) xy^2 (b) x^2y (c) $9xy^2$ (d) $3xy^2$

9. $x^{a-b} \times x^{b-c} \times x^{c-a}$ is equal to
   (a) x (b) 1 (c) 0 (d) none of these

10. The value of $\left(\frac{2p^2q^3}{3xy}\right)^0$ where p,q,x,y≠0 is equal to
    (a) 0 (b) 2/3 (c) 1 (d) none of these

11. $\{(3^3)^2 \times (4^2)^3 \times (5^3)^2\} / \{(3^2)^3 \times (4^3)^2 \times (5^2)^3\}$ is
    (a) 3/4 (b) 4/5 (c) 4/7 (d) 1

12. Which is True ?

(a) $2^0 > (1/2)^0$ (b) $2^0 < (1/2)^0$

(c) $2^0 = (1/2)^0$ (d) none of these

13. If $x^{1/p} = y^{1/q} = z^{1/r}$ and xyz = 1, then the value of p + q + r is

(a) 1 (b) 0 (c) 1/2 (d) none of these

14. The value of $y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$ is

(a) y^{a+b} (b) y (c) 1 (d) $1/y^{a+b}$

15. The True option is

(a) $x^{2/3} = \sqrt[3]{x^2}$ (b) $x^{2/3} = \sqrt{x^3}$

(c) $x^{2/3} > 3\sqrt{x^2}$ (d) $x^{2/3} < 3\sqrt{x^3}$

16. The simplified value of $16x^{-3}y^2 \times 8^{-1}x^3y^{-2}$ is

(a) 2xy (b) xy/2 (c) 2 (d) none of these

17. The value of $(8/27)^{-1/3} \times (32/243)^{-1/5}$ is

(a) 9/4 (b) 4/9 (c) 2/3 (d) none of these

18. log 6 + log 5 is expressed as

(a) log 11 (b) log 30 (c) log 5/6 (d) none of these

19. $\log_2 8$ is equal to

(a) 2 (b) 8 (c) 3 (d) none of these

20. log 32/4 is equal to

(a) log 32/log 4 (b) log 32 − log 4

(c) 23 (d) none of these

21. log (1 × 2 × 3) is equal to

(a) log 1 + log 2 + log 3 (b) log 3

(c) log 2 (d) none of these

22. The value of log 0.0001 to the base 0.1 is

(a) −4 (b) 4 (c) 1/4 (d) none of these

23. If $2 \log x = 4 \log 3$, the x is equal to

(a) 3 (b) 9 (c) 2 (d) none of these

24. $\log_{\sqrt{2}} 64$ is equal to

(a) 12 (b) 6 (c) 1 (d) none of these

25. $\log_{2\sqrt{3}} 1728$ is equal to

(a) $2\sqrt{3}$ (b) 2 (c) 6 (d) none of these

26. log (1/81) to the base 9 is equal to

(a) 2 (b) ½ (c) −2 (d) none of these

27. log 0.0625 to the base 2 is equal to

(a) 4 (b) 5 (c) 1 (d) none of these

28. Given log2 = 0.3010 and log3 = 0.4771 the value of log 6 is

(a) 0.9030 (b) 0.9542 (c) 0.7781 (d) none of these

29. The value of $\log_2 \log_2 \log_2 16$

(a) 0 (b) 2 (c) 1 (d) none of these

30. The value of log 1/3 to the base 9 is

(a) − ½ (b) ½ (c) 1 (d) none of these

31. If log x + log y = log (x+y), y can be expressed as

(a) x−1 (b) x (c) x/x−1 (d) none of these

32. The value of $\log_2 [\log_2 \{\log_3 (\log_3 27^3)\}]$ is equal to

(a) 1 (b) 2 (c) 0 (d) none of these

33. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, these x is equal to

(a) 8 (b) 4 (c) 16 (d) none of these

34. Given that $\log_{10} 2 = x$ and $\log_{10} 3 = y$, the value of $\log_{10} 60$ is expressed as

(a) x − y + 1 (b) x + y + 1

(c) x − y − 1 (d) none of these

35. Given that $\log_{10}2 = x$, $\log_{10}3 = y$, then $\log_{10}1.2$ is expressed in terms of x and y as

(a) $x + 2y - 1$ (b) $x + y - 1$

(c) $2x + y - 1$ (d) none of these

36. Given that $\log x = m + n$ and $\log y = m - n$, the value of $\log 10x/y^2$ is expressed in terms of m and n as

(a) $1 - m + 3n$ (b) $m - 1 + 3n$

(c) $m + 3n + 1$ (d) none of these

37. The simplified value of $2\log_{10}5 + \log_{10}8 - \frac{1}{2}\log_{10}4$ is

(a) 1/2 (b) 4 (c) 2 (d) none of these

38. $\log [1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$ can be written as

(a) $\log x^2$ (b) $\log x$

(c) $\log 1/x$ (d) none of these

39. The simplified value of $\log \sqrt[4]{729\sqrt[3]{9^{-1}.27^{-4/3}}}$ is

(a) log 3 (b) log 2 (c) log ½ (d) none of these

40. The value of $(\log_b a \times \log_c b \times \log_a c)^3$ is equal to

(a) 3 (b) 0 (c) 1 (d) none of these

41. The logarithm of 64 to the base $2\sqrt{2}$ is

(a) 2 (b) $\sqrt{2}$ (c) ½ (d) none of these

42. The value of $\log_8 25$ given log 2 = 0.3010 is

(a) 1 (b) 2

(c) 1.5482 (d) none of these

43. Show that $\left(\frac{x^b}{x^c}\right)^{1/bc} \times \left(\frac{x^c}{x^a}\right)^{1/ca} \times \left(\frac{x^a}{x^b}\right)^{1/ab}$ reduces to

(a) -1 (b) 0 (c) 1 (d) None

44. Show that $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$ is given by

(a) 1 (b) -1 (c) 0 (d) 3

45. On simplification $\frac{1}{1+z^{a-b}+z^{a-c}}+\frac{1}{1+z^{b-c}+z^{b-a}}+\frac{1}{1+z^{c-a}+z^{c-b}}$ would reduces to

(a) 0 (b) -1 (c) 1 (d) 2

46. If $x=5^{1/3}+5^{-1/3}$ prove that $5x^3-15x$ is given by

(a) 25 (b) 26 (c) 27 (d) 30

47. On simplification $\frac{x^{\frac{a}{a-b}}}{x^{\frac{a}{a+b}}} \times \frac{x^{\frac{b}{b-a}}}{x^{\frac{b}{b+a}}}$ reduces to

(a) 1 (b) -1 (c) 0 (d) None

48. On simplification $\left[\frac{x^{ab}}{x^{a^2+b^2}}\right]^{a+b} \times \left[\frac{x^{b^2+c^2}}{x^{bc}}\right]^{b+c} \times \left[\frac{x^{ca}}{x^{c^2+a^2}}\right]^{c+a}$ reduces to

(a) x^{-2a3} (b) x^{2a3} (c) $x^{-2(a^3+b^3+c^3)}$ (d) $x^{2(a^3+b^3+c^3)}$

49. If $a=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}$ then the value of a^3+3a-2 is

(a) 3 (b) 0 (c) 2 (d) 1

50. If $a=\frac{1}{2}\left(5-\sqrt{21}\right)$ then the value of $a^3+a^{-3}-5a^2-5a^{-2}+a+a^{-1}$ is

(a) 0 (b) 1 (c) 5 (d) - 1

51. $\frac{1}{1+\log_a(bc)}+\frac{1}{1+\log_b(ca)}+\frac{1}{1+\log_c(ab)}$

(a) 0 (b) 1 (c) 3 (d) -1

52. If $\frac{\log a}{y-z}=\frac{\log b}{z-x}=\frac{\log c}{x-y}$ the value of abc is

(a) 0 (b) 1 (c) -1 (d) None

53. If $\frac{1}{2}\log a=\frac{1}{3}\log b=\frac{1}{5}\log c$ the value of a^4-bc is

(a) 0 (b) 1 (c) -1 (d) None

54. If $\frac{1}{4}\log_2 a=\frac{1}{6}\log_2 b=-\frac{1}{24}\log_2 c$ the value of a^3b^2c is

(a) 0 (b) 1 (c) -1 (d) None

55. If $\frac{1}{\log_a t}+\frac{1}{\log_b t}+\frac{1}{\log_c t}=\frac{1}{\log_z t}$ then the value if z is given by

(a) abc (b) a+b+c (c) a(b+c) (d) (a+b)c

56. If $l = 1+\log_a bc$, $m = 1+\log_b ca$, $n = 1+\log_c ab$ then the value of $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} - 1$ is

(a) 0 (b) 1 (c) -1 (d) 3

57. If $(4.8)^x = (0.48)^y = 1{,}000$ then the value of $\frac{1}{x} - \frac{1}{y}$ is

(a) 3 (b) -3 (c) 1/3 (d) -1/3

58. If $x^{2a-3}y^{2a} = x^{6-a}\, y^{5a}$ then the value of $a\log\left(\frac{x}{y}\right)$ is

(a) 3 log x (b) log x (c) 6 log x (d) 5 log x

HOMEWORK SOLUTION

1. (c) $\frac{4}{x^{1/4}}$

2. (c) 2

3. (c) 4

4. (b) 2

5. (a)

6. (a) 1

7. (b) A positive integer

$2^{1/2}.(4)^{3/4} = 2^{1/2}.(2^2)^{3/4}$

$= 2^{1/2}.(2)^{3/2}$

$= 2^{\frac{1}{2}+\frac{3}{2}}$

$= 2^2$

$= 4$

8. (d) $3xy^2$

$\left(\frac{81.x^4}{y-8}\right)^{1/4} = \left(\frac{3^4.x^4}{y^{-8}}\right)^{1/4}$

$= \frac{3x}{y^{-2}} = 3xy^2$

9. (b) 1 Cyclical terms (in power)

10. (c) 1

11. (d) 1

$\frac{3^6 \times 4^6 \times 5^6}{3^6 \times 4^6 \times 5^6} = 1$

12. (c) $2^\circ = \left(\frac{1}{2}\right)^o$

13. (b) 0

$x^{1/p} = y^{1/q} = z^{1/r} = k$

$\therefore\ x^{1/p} = k \quad \Rightarrow x = k^p$

$\therefore\ y^{1/q} = k \quad \Rightarrow x = k^q$

$\therefore\ z^{1/r} = k \quad \Rightarrow z = k^r$

$xyz = 1$

$\therefore\ k^p . k^q . k^r = k^o$

$\therefore\ k^{p+q+r} = k^o$

$\square\ p + q + r = 0$

14. (d) $\frac{1}{y^{a+b}}$

$y^{a-b} \times y^{b-c} \times y^{c-a} \times y^{-a-b}$

$= y^{a-b+b-c+c-a-a-b}$

$= y^{-(a+b)}$

$= \frac{1}{y^{a+b}}$

15. (a) $x^{2/3} = \sqrt[3]{x^2}$

16. (c) 2

$= 16 . x^{-3} . y^2 . 8^{-1} . x^3 . y^{-2}$

$= \frac{16 . y^2 . x^3}{8 . x^3 . y^2}$

$= 2$

17. (a) $\frac{9}{4}$

$\left(\frac{8}{27}\right)^{-1/3} . \left(\frac{32}{243}\right)^{-1/5} = \left(\frac{27}{8}\right)^{1/3} . \left(\frac{243}{32}\right)^{1/5}$

$= \frac{3}{2} . \frac{3}{2}$

$= \frac{9}{4}$

18. (b) log 30

log (6 × 5) = log 30

19. (c) 3

$\log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3$

20. (b) log 32 – log 4

21. (a) log 1 + log 2 + log 3

22. (b) 4

$\log_{0.1} 0.0001 = \log_{0.1} (0.1)^4 = 4 \log_{0.1} 0.1 = 4$

23. (b) 9

$\text{Log } x^2 = \log 3^4$

$\square X^2 = 3^4$

$\square X = 3^2 = 9$

24. (a) 12

$\text{Log}_{\sqrt{2}} 64 = \log_{\sqrt{2}} \left(\left(\sqrt{2}\right)^2\right)^6$

$= \log_{\sqrt{2}} \left(\sqrt{2}\right)^{12}$

$= 12$

25. (c) 6

$\text{Log}_{258} 1728 = \log_{2\sqrt{3}} \left(2\sqrt{3}\right)^6$

$= 6$

26. (c) -2

$\text{Log}_9 \left(1/81\right) = \log_9 (81)^{-1}$

$= \log_9 (9)^{-2}$

$= -2$

27. (d) None of these trial and error

28. (c) 0.7781

$\therefore$ Log 6 = log (2 × 3) = log 2 + log 3

= 0.3010 + 0.4771

= 0.7781

29. (c) 1

$\log_2 \log_2 \log_2 16 = \log_2 \log_2 \log_2 2^4$

$= \log_2 \log_2 4$

$= \log_2 \log_2 2_2$

$= \log_2 2$

$= 1$

30. (a) $\frac{-1}{2}$

$\log_9 \frac{1}{3} = \frac{\log(3)^{-1}}{\log 9} = \frac{-1 \log 3}{2 \log 3} = \frac{-1}{2}$

31. (c) $\frac{x}{x-1}$

$\log x + \log y = \log (x + y)$

$\square \log (xy) = \log (x + y)$

$\square xy = x + y$

$\square xy - y = x$

$\square y (x - 1) = x$

$\therefore y = \frac{x}{x-1}$

32. (c) 0

$\log_2 [\log_2 \{\log_3 (\log_3 27^3)\}]$

$= \log_2 [\log_2 \{\log_3 (\log_3 (3^3)^3)\}]$

$= \log_2 [\log_2 \{\log_3 9\}]$

$= \log_2 [\log_2 2]$

$= \log_2 1$

33. (a) 8

$\text{Log}_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\therefore \frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = \frac{21}{4}$

$\therefore \frac{\log x}{\log 2}\left[1 + \frac{1}{2} + \frac{1}{4}\right] = \frac{21}{4}$

$\therefore \log_2 x \left[\frac{4+2+1}{4}\right] = \frac{21}{4}$

$\therefore \log_2 x = 3$

$\therefore 2^3 = x$

$\therefore x = 8$

34. (b) x + y + 1

$\log_{10} 60 = \log_{10} (2 \times 3 \times 10)$

$= \log 2 + \log 3 + \log 10$

$= x + y + 1$

35. (c) 2x + y - 1

$\log_{10} 12 = \log_{10} \frac{12}{10}$

$= \log 12 - \log 10$

$= \log (2 \times 2 \times 3) - \log 10$

$= \log 2 + \log 2 + \log 3 - \log 10$

$= x + x + y - 1$

$= 2x + y - 1$

36. (a) 1 − m + 3n

$\log \frac{10x}{y^2} = \log 10 + \log x - \log y^2$

$= \log 10 + \log x - 2 \log y$

$= 1 + m + n - 2 (m - n)$

$= 1 + m + n - 2m - 2n$

$= 1 - m + 3n$

37. (c) 2

$$2\log_{10}5 + \log_{10}8 - \log_{10}4$$

$$= \log_{10}5^2 + \log_{10}8 - \log_{10}4^{\frac{1}{2}}$$

$$= \log_{10}\left(\frac{25 \times 8}{2}\right)$$

$$= \log_{10}100$$

$$= 2$$

38. (b) log x

$$= \log\left[1 - \{1 - (1 - x^2)^{-1}\}^{-1}\right]^{\frac{-1}{2}}$$

$$= \log\left[1 - \{1 - \frac{1}{1 - x^2}\}^{-1}\right]^{\frac{-1}{2}}$$

$$= \log\left[1 - \{\frac{1 - x^2 - 1}{1 - x^2}\}^{-1}\right]^{\frac{-1}{2}}$$

$$= \log\left[1 - \{\frac{x^2}{1 - x^2}\}^{-1}\right]^{\frac{-1}{2}}$$

$$= \log\left[\frac{1}{x^2}\right]^{\frac{-1}{2}}$$

$$= \log (x^{-2})^{\frac{-1}{2}}$$

$$= \log x$$

39. (a) log 3

$$\log \sqrt[4]{729\sqrt[3]{9^{-1}(27)^{\frac{-4}{3}}}}$$

$$= \log\left[729\,(9^{-1}\,(27)^{\frac{-4}{3}})^{\frac{1}{3}}\right]^{\frac{1}{4}}$$

$$= \log\left[3^6\,(3^{-2} \times (3^3)^{\frac{-4}{3}})^{\frac{1}{3}}\right]^{\frac{1}{4}}$$

$$= \log\left[3^6\,(3^{-2\,-4})^{\frac{1}{3}}\right]^{\frac{1}{4}}$$

$$= \log\left[3^6\,3^{-2}\right]^{\frac{1}{4}}$$

$$= \log\left[3^4\right]^{\frac{1}{4}}$$

$$= \log 3$$

40. (c) 1

The value of $(\log_b a \times \log_c b \times \log_a c)$

$$= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right)^3$$

$$= (1)^3$$

$$= 1$$

41. (d) 4

$$\log_{2\sqrt{2}} 64 = \log_{2\sqrt{2}} (2\sqrt{2})^4$$

$$= 4$$

42. (c) 1.5482

$$\log_8 25 = \log_8 \left(\frac{25 \times 4}{4}\right) = \log_8 \left(\frac{100}{4}\right)$$

$$= \log_8 \left(\frac{100}{4}\right)$$

$$= \frac{\log 100 - \log 4}{\log 8}$$

$$= \frac{\log 20 - \log 2}{\log 2}$$

43. Show that $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot$

M – I $\left(x^{b-c}\right)^{\frac{1}{bc}} \cdot \left(x^{c-a}\right)^{\frac{1}{ca}} \cdot \left(x^{a-b}\right)^{\frac{1}{ab}} \cdot$

$$= x^{\frac{b-c}{bc} + \frac{c-a}{ca} + \frac{a-b}{ab}}$$

$$= x^{\frac{(b-c)+b(c-a)+c(a-b)}{abc}}$$ [Taking L.C.M. in the power]

$$= x^{\frac{ab-ac+bc-ac+ac-ab}{abc}}$$

$$= x^{\frac{0}{abc}}$$

$$= x^\circ = 1$$

M – II $= \frac{x^{\frac{b}{bc}}}{x^{\frac{c}{bc}}} \frac{x^{\frac{c}{ca}}}{x^{\frac{a}{ca}}} \frac{x^{\frac{a}{ab}}}{x^{\frac{b}{ab}}}$

$$= \frac{x^{\frac{1}{c}}}{x^{\frac{1}{b}}} \frac{x^{\frac{1}{a}}}{x^{\frac{1}{c}}} \frac{x^{\frac{1}{b}}}{x^{\frac{1}{a}}} = 1$$

44. S.T $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2}\left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2}\left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

$= x^{(a-b)(a2+ab+b2)} \cdot x^{(b-c)(b2+bc+c2)} \cdot x^{(c-a)(c2+ac+a2)}$

$= x^{a3-b3}\, x^{b3-c3} \cdot x^{c3-a3}$

$= x^{a3-b3+b3-c3+c3-a3}$

$= x^{\circ} = 1$

45. on simplification $\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$

Multiply 1st term by Z^{-a} (both no and Dr.)

2nd term by Z^{-b} (_____,_____)

3rd term by Z^{-c} (_____,_____)

$$= \frac{z^{-a}}{z^{-a}+z^{-b}+z^{-c}} + \frac{z^{-b}}{z^{-b}+z^{-c}+z^{-a}} + \frac{z^{-c}}{z^{-c}+z^{-a}+z^{-b}}$$

$$= \frac{z^{-a}+z^{-b}+z^{-c}}{z^{-a}+z^{-b}+z^{-c}} = 1$$

46. $x = 5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$ find $5x^3 - 15x = ?$

$x^3 = \left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)^3$ [Cubing on the sides]

$x^3 = \left(5^{\frac{1}{3}}\right)^3 + \left(5^{\frac{-1}{3}}\right)^3 + 3\left(5^{\frac{1}{3}}\right) x\, 3\left(5^{\frac{-1}{3}}\right) x \left(5^{\frac{1}{3}} + 5^{\frac{-1}{3}}\right)$

[$\because (a+b)^2 - a^3 + b^3 - 3ab(a+b)$

$x^3 = 5 + 5^{-1} + 3(5^{\circ})(x)$ [$\because x = 5^{\frac{1}{3}} + 5^{\frac{-1}{3}}$]

$x^3 = 5 + \frac{1}{5} + 3x$

$x^3 - 3x = \frac{26}{5}$

$\square 5x^3 - 15x = 26$

47. a. $\left[\frac{x^{\frac{a}{a-b}}}{x^{\frac{a}{a+b}}} \times \frac{x^{\frac{b}{b-a}}}{x^{\frac{b}{b+a}}}\right]$

$$= \left[\frac{x^{\frac{a}{a-b}-\frac{b}{a-b}}}{x^{\frac{a}{a+b}+\frac{b}{b+a}}}\right] = \left[\frac{x^{\frac{a-b}{a-b}}}{x^{\frac{a+b}{a+b}}}\right] = \frac{x}{x} = 1$$

48. $\left(\frac{x^{ab}}{x^{a^2+b^2}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{ca}}{x^{c^2+a^2}}\right)^{a+b}$

$= (x^{ab-a^2-b^2})^{a+b}\ x^{(b^2+c^2-bc)(b+c)}\ x^{(ca-c^2-a^2)(c+a)}$

$= x^{-(a^2+b^2-ab)(a+b)}\ x^{(b^2+c^2-bc)(b+c)}\ x^{-(c^2-a^2-ca)(c+a)}$

$= x^{-(a^3+b^3)}\ x^{b^3+c^3}\ x^{-(c^3+a^3)}$

$= x^{-a^3-b^3+b^3+c^3-c^3-a^3}$

$= x^{-2a^3}$

49. If a = $a = \sqrt[3]{\sqrt{2}+1} - \sqrt[3]{\sqrt{2}-1}$

Then the value of $a^3 + 3a - 2$ is

$a = (\sqrt{2}+1)^{\frac{1}{3}} - (\sqrt{2}+1)^{\frac{1}{3}}$

Cubing on both sides, we get,

$a^3 = \left[(\sqrt{2}+1)^{\frac{1}{3}} - (\sqrt{2}-1)^{\frac{1}{3}}\right]^3$

$= \cancel{\sqrt{2}} + 1 - (\cancel{\sqrt{2}} - 1) - 3\left[(\sqrt{2}+1)^{\frac{1}{3}}\left(\sqrt{2}-1\right)^{\frac{1}{3}}\right](\sqrt{2}+1)^{\frac{1}{3}} - (\sqrt{2}-1)^{\frac{1}{3}}$

$= \sqrt{2} + 1 - (\sqrt{2} - 1) - 3\left[(\sqrt{2}+1)^{\frac{1}{3}}\left(\sqrt{2}-1\right)^{\frac{1}{3}}\right](a)$

$= \cancel{\sqrt{2}} + 1 - \cancel{\sqrt{2}} + 1 - 3a\left[(\sqrt{2})^2 - (1)^2\right]^{\frac{1}{3}}$

$a^3 = 2 - 3a[2-1]^{\frac{1}{3}}$

$a^3 = 2 - 3a$

$\therefore a^3 + 3a - 2 = 0$

50. $a = \frac{1}{2}(5 - \sqrt{21})$

$a = \frac{(5-\sqrt{21})}{2}; \frac{1}{a} = \frac{1}{2}(5+\sqrt{21})$

$\therefore a + \frac{1}{a} = \frac{5\sqrt{21}+5+\sqrt{21}}{2} = \frac{10}{2} = 5$

51. $\frac{1}{1+\log_a bc}+\frac{1}{1+\log_b ca}+\frac{1}{1+\log_c ab}=?$

M – I

Let, a = 2, b = 4, c = 8

$=\frac{1}{1+\log_2 32}+\frac{1}{1+\log_4 16}+\frac{1}{1+\log_8 8}$

$=\frac{1}{1+5\log_2 2}+\frac{1}{1+2\log_4 4}+\frac{1}{1+1}$

$=\frac{1}{1+5}+\frac{1}{1+2}+\frac{1}{2}$

$=\frac{1}{6}+\frac{1}{3}+\frac{1}{2}$

$=\frac{1+2+3}{6}$

$=\frac{6}{6}$

M-II

$\frac{1}{\log_a a+\log_a bc}+\frac{1}{\log_b b+\log_b ac}+\frac{1}{\log_c c+\log_a ab}$

$=\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}$

$=\log_a^{abc}+\log_b^{abc}+\log_c^{abc}$

$=\log_{abc} a+\log_{abc} b+\log_{abc} c$

$=\log_{abc} abc=1$

52. $\frac{\log a}{y-z}=\frac{\log b}{z-x}=\frac{\log c}{x-y}=k$

$\log a = k(y-z)$

$\log b = k(z-x)$

$\log c = k(x-y)$

$\log a+\log b+\log c=0$

$\log abc = 0 = \log 1$

$\Rightarrow abc = 1$

Hence, x, y, z are in cyclic order, $\Rightarrow$ their sums is 0

M-II

$\log_{10} abc = 0$

$10^0 = abc = 1$

53. $a^4 - bc = ?$

$\frac{1}{2}\log a = \frac{1}{3}\log b = \frac{1}{5}\log c = k$

$\log_{10} a = 2k ; \log_{10} b = 3k ; \log_{10} c = 5k$

$10^{2k} = a ; 10^{3k} = b ; 10^{5k} = c$

$a^4 = 10^{8k}$;

$10^{8k} - 10^{3k} \times 10^{5k}$

$10^{8k} - 10^{8k}$

$= 0$

54. $\frac{1}{4}\log_2 a = \frac{1}{6}\log_2 b = = \frac{-1}{24}\log_2 c = k \qquad a^3 b^2 c = ?$

$\log_2 a = 4k ; \log_2 b = 6k ; \log_2 c = -24k$

$2^{4k} = a ; \quad 2^{6k} = b ; \quad 2^{-24k} = c$

$a^3 = (2^{4k})^3 = 2^{12k} \quad ; \; b^2 = (2^{6k})^2 = 2^{12k}$

$\therefore a^3 b^2 c = 2^{12k} \quad 2^{12k} \; 2^{-24k}$

$= 2^{24k - 24k}$

$= 2^o = 1$

55. $\frac{1}{\log_a t} + \frac{1}{\log_b t} + \frac{1}{\log_c t} = + \frac{1}{\log_z t} \qquad z = ?$

$\log_t a + \log_t b + \log_t c = \log_t z$

$\log_t abc \; = \log_t z$

$abc = z$

56. $L = 1 + \log_a bc$

$M = 1 + \log_b ac$

$M = 1 + \log_c ab$

M- I

Take, $a = 2 ; b = 2^2 ; c = 2^3 = 8$

$L = 1 + \log_2 {}^{32} = 1 + 5 \log_2 {}^{2} = 6$

$M = 1 + \log_4 {}^{16} = 1 + 2 = 3$

$N = 1 + \log_8 {}^{8} = 1 + 1 = 2$

$= \frac{1}{L} + \frac{1}{M} + \frac{1}{N} - 1$

$= \frac{1}{6} + \frac{1}{3} + \frac{1}{2} - 1$

$= \frac{1+2+3}{6} - 1$

$= 1 - 1 = 0$

M- II

Let a = b = c

$L = 1 + \log_a bc = 1 + \log_a a^2 = 1 + 2 = 3$

$M = 3 ; N = 3.$

$= \frac{1}{L} + \frac{1}{M} + \frac{1}{N} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - 1 = 1 - 1 = 0$

57. $(4.8)^x = (0.48)^y = 1{,}000$ $\qquad \frac{1}{x} + \frac{1}{y} = ?$

$(4.8)^x = (0.48)^y = (10)^3$

$(4.8)^x = 10^3 \qquad ; (0.48)^y = 10^3$

$4.8 = 10^{\frac{3}{x}} \qquad ; 0.48 = 10^{\frac{3}{y}}$

$0.48 \times 10 = 10^{\frac{3}{y}} 10$

$4.8 = 10^{\frac{3}{y}+1}$

$\therefore 10^{\frac{3}{y}} = 10^{\frac{3}{y}+1}$

$\therefore \frac{3}{x} = \frac{3}{y} + 1$

$\therefore \frac{3}{x} - \frac{3}{y} = 1$

$= 3\left(\frac{1}{x} - \frac{1}{y}\right) = 1$

$= \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

58. $x^{2a-3} y^{2a} = x^{6-a} y^{5a}$ $\quad a\log\left(\frac{x}{y}\right) = 3\log^{x}$

$$\frac{x^{2a-3}}{x^{6-a}} = \frac{y^{5a}}{y^{2a}}$$

$$x^{2a-3-6} = y^{3a}$$

$$x^{30-9}\, y^{3a}$$

$$\frac{x^{3a}}{x^{9}} = y^{3a}$$

$$\frac{x^{3a}}{y^{3a}} = x^{9}$$

$$\left(\frac{x}{y}\right)^{3a} = x^{9}$$

$$\cancel{3}a \log \frac{x}{y} = \cancel{9}^{3} \log x$$

SELF ASSESSMENT TEST 1
Ratio, Proportion and Mixtures
20 Question, 20 Marks

1. The ratio of A to B is 4 : 5 and that of C to B is 3 : 2. If A = 800, C = ?
   a) 1000 b) 1200 c) 1500 d) 2000

2. Three numbers A, B and C are in the ratio 1 / 2 : 2 / 3 : 3 / 4. The difference between the greatest and the smallest number is 36. Find A.
   a) 60 b) 72 c) 84 d) None of the above

3. Ratio of land and water on earth is 1 : 2. In northern hemisphere, the ratio is 2 : 3. What is the ratio in the southern hemisphere?
   a) 3 : 11 b) 2 : 11 c) 4 : 11 d) 5 : 11

4. The ratio of copper and zinc in a 63 kg alloy is 4 : 3. Some amount of copper is extracted from the alloy and the ratio becomes 10 : 9. How much copper is extracted?
   a) 6 kg b) 8 kg c) 12 kg d) 10 kg

5. A bag contains Rs. 55 in the denominations of Re 1, 50 paise and 25 paise coins. The coins are in the ratio 1 : 2 : 3. Find the number of 50 paise coins.
   a) 15 b) 30 c) 40 d) 45

6. A person cover certain distance by train, bus and car in the ratio 4 : 3 : 2. The ratio of fare is 1 : 2 : 4 per km. The total expenditure as fare is Rs. 720. Find the total expenditure as fare on train.
   a) Rs. 140 b) Rs. 150 c) Rs. 160 d) Rs. 170

7. The ratio of the expenditure of A, B and C are 16 : 12 : 9 respectively and their savings are 20%, 25% and 40% of their incomes. If the sum of their income is Rs. 15,300; find B's salary.
   a) Rs. 4800 b) Rs. 5000 c) Rs. 4900 d) Rs. 5100

8. One year ago, the ratio between A and B salary was 3 : 5. The ratio of their individual salaries of last year and present year are 2 : 3 and 4 : 5 respectively. If their total salary for the present year is Rs. 4300, find the present salary of A.

a) Rs. 1200 b) Rs. 1800 c) Rs. 1600 d) Can't be determined

9. The income of A and B is in the ratio 5 : 3. The expenses of A, B and C are in the ratio 8 : 5 : 2. If C spends Rs. 2000 and B saves Rs. 700, then how much did A saves?

a) Rs. 500 b) Rs. 1500 c) Rs. 1000 d) Rs. 250

10. The ratio of total amount distributed in all the males and females as salary is 6 : 5. The ratio of salary of each male and female is 2 : 3. Find the ratio of the number of male and female.

a) 5 : 9 b) 5 : 7 c) 9 : 5 d) 7 : 5

11. Rs. 56000 is to be divided among A, B, C and D in such a way that the ratio of share of A : B is 1 : 2, B : C is 3 : 1 and C : D is 2 : 3. Find the sum of share of A & C and B & C.

a) Rs. 24000, Rs. 30000 b) Rs. 20000, Rs. 32000

c) Rs. 24000, Rs. 32000 d) Rs. 20000, Rs. 30000

12. A, B and C works on a project for 30, 50 and 60 days respectively. The ratio of the salary of each day is 4 : 3 : 2 respectively. If the total amount received by A is Rs. 14,400, find total amount received by B.

a) Rs. 18000 b) Rs. 19000 c) Rs. 18500 d) Rs. 19500

13. Two numbers are in the ratio 4 : 5. If each number is reduced by 25, the ratio becomes 3 : 4. Find the second number.

a) 120 b) 130 c) 125 d) 135

14. The price of gold is directly proportional to square of its weight. A person broke down the gold in the ratio of 3 : 2 : 1 and faces a loss of Rs. 46,200. Find the initial price of the gold.

a) Rs. 75,200 b) Rs. 75,400 c) Rs. 75,300 d) Rs. 75,600

15. Rs. 78,000 is distributed among A, B and C such that the share of A = 3 / 4 share of B and share of B = 2 / 3 of the share of C. What is the difference between the shares of B and C?

a) Rs. 9000 b) Rs. 10000 c) Rs. 11000 d) Rs. 12000

16. A dog chases a rabbit. The dog takes 6 leaps for every 7 leaps of the rabbit. The rabbit takes 6 leaps for every 5 leaps of the dog. What is the ratio of speed of dog and rabbit?

a) 36 : 35 b) 36 : 40 c) 35 : 36 d) None of the above

17. A mixture contains milk and water in the ratio of 4 : 3 respectively. If 6 litres of water is added to this mixture, the respective ratio of water and milk becomes 7 : 8. What is the quantity of milk in the original mixture?

a) 96 litres b) 84 litres c) 36 litres d) 48 litres

18. There are X members in a club, whose average age is 26 years. 3 more persons join them at the 35th Annual General Meeting, and thus the average age of members now increases by 1 year. If the average age of 3 new members joining at the 35th AGM is 29 years, find X.

a) 6 b) 7 c) 8 d) None of the above

19. If the ratio of volume of two cubes with Amit and Ajay is 125 : 8, then find the ratio of the total surface area of the cubes with Ajay and Amit.

a) 25 : 4 b) 4 : 25 c) 5 : 2 d) 2 : 5

20. Mr. Azon divides $ 51,300 among his four partners A, B, C and D such that 3 times A's share = 4 times B's share = 5 times C's share = 6 times D's share. What is the share of B?

a) $ 11,000 b) $ 13,500 c) $ 15,300 d) $ 12,350

EXPLANATORY ANSWERS

1. A : B = 4 : 5
   B : C = 2 : 3
   A : B : C = 8 : 10 : 15
   C = 800/8 * 15 = 1500
   Option C

2. A : B : C = 1 / 2 : 2 / 3 : 3 / 4 = 6 : 8 : 9
   9x – 6x = 3x = 36; x = 12
   A = 6 * 12 = 72
   Option B

3. Let total = 300. At earth, Land = 100, Water = 200
   In northern hemisphere = 150, Land = 150 * 2/5 = 60, Water = 90
   In southern hemisphere: Land = 100 – 60 = 40, Water = 200 – 90 = 110
   Ratio = 40 : 110 = 4 : 11
   Option C

4. Alloy: Copper = 4/7 * 63 = 36; Zinc = 3/7 * 63 = 27
   (36 – X) / 27 = 10 / 9
   36 – X = 30; X = 6
   Option A

5. X + (2X)*1/2 + (3X)*1/4 = 55
   11X/4 = 55
   X = 20
   No. of 50 paise coins = 2X = 40
   Option C

6. 4x * 1 + 3x * 2 + 2x * 4 = 18x = 720; x = 40
   Fare expenditure on train = 4 * 40 = 160; Option C

7. Expenditure = 100% - Savings%
   16x/80% + 12X/75% + 9X/60% = 20x + 16x + 15x = 51x = 15300; x = 300
   B's income = 16 * 300 = 4800; Option A

8. One year ago = 3x, 5x
Present salary = (3x)3/2 : (5x)5/4 = 18 : 25
A's present salary = 18/43 * 4300 = 1800; Option B

9. Income = 5x, 3x
Expenses = 8y, 5y, 2y
2y = 2000; y = 1000
3x – 5000 = 700; x = 1900
A saves = (5 * 1900) – (8 * 1000) = 9500 – 8000 = 1500
Option B

10. Person = Total salary/Salary per person
M : F = 6/2 : 5/3 = 9 : 5
Option C

11. A : B = 1 : 2
B : C = 3 : 1
C : D = 2 : 3
A : B : C : D = 6 : 12 : 4 : 6 = 3 : 6 : 2 : 3
A & C = 5/14 * 56000 = 20000
B & C = 8/14 * 56000 = 32000
Option B

12. 30x(4) + 50x(3) + 60x(2) = 390x
120x = 14400; x = 120
B's amount = 150x = 150 * 120 = 18000
Option A

13. (4x – 25) : (5x – 25) = 3 / 4
16x – 100 = 15x – 75
Or, x = 25
Second number = 5 * 25 = 125
Option C

14. $P = k.w^2$

New $P = k (9 + 4 + 1) = 14k$

Old $P = k(6)2 = 36k$

Loss $= 36k - 14k = 22k = 46200$; $k = 2100$

Old $P = 36 * 2100 = 75600$; Option D

15. A : B = 3 : 4

B : C = 2 : 3

A : B : C = 6 : 8 : 12

$6x + 8x + 12x = 78000$; $x = 78000 / 26 = 3000$

B – C difference $= 4 * 3000 = 12000$

Option D

16. 1 leap of dog = 1 units

6 leaps of dog = 6 units = 7 leaps of rabbit; 1 leap of rabbit = 6/7 units

In a given time: 6 rabbit = 5 dog; 6(6/7) : 5 (1) = 36 : 35

Speed of dog and rabbit = 35 : 36

Option C

17. $3x/7 + 6 = 7/15 (x + 6)$

$45x + 630 = 49x + 294$

$4x = 336$, $x = 84$

Milk in original mixture $= 4/7 * 84 = 48$

Option D

18. Total age of 26 members = 26X

Total age of 29 members = $26X + (29x3) = 27(X + 3)$

$26X + 87 = 27X + 81$

$X = 6$

Option A

19. Amit : Ajay

V = 125 : 8

Side = 5 : 2

Total Surface = 25 : 4

Option B

20. A : B : C : D = 1 / 3 : 1 / 4 : 1 / 5 : 1 / 6 = 120 : 90 : 72 : 60

B's share = 90 / 342 * 51300 = 13500

Option B

SELF ASSESSMENT TEST 2
Surds, Indices & Logarithms
30 Question, 30 Marks

1. The value of is: $\sqrt{30+\sqrt{30+\sqrt{30+.....to\infty}}}$

   a) 6 b) 5 c) 3 d) None of these

2. If $8^x = 4^y$, what is the value of $K^{(3x-2y)}$?

   a) K b) K2 c) 0 d) 1

3. If $X = \log_2\log_3\log_2 512$; find the value of (X − 1)!

   a) 1 b) 4 c) 6 d) 24

4. If $2\log\frac{4}{3}-\log\frac{x}{10}+\log\frac{63}{160}=0$, find the value of x.

   a) 3 b) 4 c) 7 d) 9

5. If log 2 = 0.30103; then find the number of digits in 25^{10}

   a) 13 b) 15 c) 14 d) 25

6. Find the value of $9^{1/3}.9^{1/9}.9^{1/27}.9^{1/81}$.

   a) 3 b) 9 c) 81 d) None of the above

7. If X = cube root of 2, Y = 6th root of 3, Z = 9th root of 4; then which of the following is true?

   a) X = Y = Z b) X > Y > Z c) X < Y < Z d) X = Y < Z

8. If $(AB)^{1/2}$ = 6 and A & B are positive integers, then which of the following could not be a value of C = (A − B)?

   a) 0 b) 5 c) 8 d) 9

9. If $x = 3 + (8)^{1/2}$, then what is the value of $x^4 + x^{-4}$?

   a) 1154 b) 1145 c) 1164 d) 1146

10. If $a^z = b^y = c^x$ and $b^2 = ca$, then find the value of y(x + z).

    a) 2 b) 2xz c) xz d) None of these

11. If $\log\left(\frac{a+b}{7}\right)=\frac{1}{2}(\log a+\log b)$, then which of the following is true?

a) a + b = 49 b) $a^2 + b^2 = 47ab$

c) $a^2 - b^2 = 49ab$ d) None of these

12. Given $X = \frac{1}{6}\cdot\sqrt{\frac{3\log(1728)}{1+\frac{1}{2}\log(0.36)+\frac{1}{3}\log 8}}$, find the value of (2X − 1).

a) 4 b) 1 c) 0 d) − 1

13. If $x=\frac{6ab}{a+b}$, then the value of $\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}$ is equal to:

a) 0 b) 1 c) 2 d) None of the above

14. If $X = 3\sqrt{2} + 7\sqrt{8} + \sqrt{27} + 5\sqrt{3}$. Find X.

a) $17\sqrt{2} + \sqrt{3}$ b) $17\sqrt{2} + 8\sqrt{3}$

c) $15\sqrt{2} + 8\sqrt{3}$ d) None of the above

15. Given $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$. The value of x is:

a) 1 b) 331/5 c) 263/20 d) 17/21

16. If a : b = 3 : −1, find $\frac{ab}{a+b}$

a) 1 b) 3 c) − 3 d) None of the above

17. If $\log\frac{12}{13}-\log\frac{7}{25}+\log\frac{91}{3}=x$, find the value of x.

a) 0 b) 1 c) 2 d) 3

18. $\frac{1}{1+\frac{1}{x}}$ is equal to which of the following?

a) (x + 1) b) $(x + 1)^{-1}$

c) $x(x + 1)^{-1}$ d) None of the above

19. If $\frac{a^3-b^3}{a^3+b^3}=\frac{13}{14}$; then find the value of $\frac{a+b}{a-b}$.

a) 1 b) 2 c) 1.5 d) None of the above

20. If $(x + x^{-1}) = 3$, then the value of $(x^6 + x^{-6})$ is

a) 927 b) 364 c) 414 d) 322

21. Which of the following relations is true?

a) $\sqrt{4} + \sqrt{3} < \sqrt{5} + \sqrt{2}$

b) $\sqrt{4} + \sqrt{3} = \sqrt{5} + \sqrt{2}$

c) $\sqrt{4} + \sqrt{3} > \sqrt{5} + \sqrt{2}$

d) None of the above is true

22. The expression simplifies to: $\frac{(y-1)(y-2)(y^2-9y+14)}{(y-7)(y^2-3y+2)}$

a) (y − 1) b) (y − 7)

c) (y − 2) d) $(y - 7)^{-1}$

23. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then find the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}$.

a) 13/15 b) 15/13

c) 2/13 d) 11/13

24. Find the value of $\frac{1}{\sqrt{11-2\sqrt{30}}} - \frac{3}{\sqrt{7-2\sqrt{10}}} - \frac{4}{\sqrt{8+4\sqrt{3}}}$

a) $\sqrt{6} + \sqrt{2}$ b) $\sqrt{6} + \sqrt{5}$

c) $\sqrt{5} + \sqrt{2}$ d) None of the above

25. If X = 3 log5 + 2 log4 − log2, Find the value of (X + 3).

a) 0 b) 6 c) 3 d) None of the above

26. If logX = log1.5 + log12, Find the value of X/3.

a) 0 b) 6 c) 3 d) None of the above

27. Find the value of X, if: Log (X − 13) + 3 Log2 = Log(3X + 1).

a) 21 b) 22 c) 20 d) 24

28. If log3 = 0.4771, find the value of $\log (0.81)^2 * \log (27/10)^{2/3} \div \log 9$.

a) 2.702 b) − 0.0552 c) 2.2402 d) − 2.689

29. $E = \frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}$. Find the value of (E – 3).

a) 0 b) 1 c) – 1 d) None of the above

30. If $\log\frac{75}{35} + 2\log\frac{7}{5} - \log\frac{105}{x} - \log\frac{13}{25} = 0$ find the value of x.

a) 13 b) 45 c) 50 d) 65

EXPLANATORY ANSWERS

1. 30 = 6*5. If A = K(K +1) and you find "+" sign in it, answer is always (k + 1). Option A

2. $8^x = 4^y$

   3x = 2y

   (3x − 2y) = 0. $K^0 = 1$; Option D

3. $X = \log_2\log_3\log 2512$

   $= \log_2\log_3\log 2(2)^9$

   $= \log_2\log^3 9$

   $= \log_2\log_3(3)^2$

   $= \log_2 2 = 1$

   (X − 1)! = 0! = 1; Option A

4. $2\log\frac{4}{3}-\log\frac{x}{10}+\log\frac{63}{160}=0$

   $2\log\frac{4}{3}+\log\frac{63}{160}=\log\frac{x}{10}$

   $\log\left(\frac{4*4*7*3*3}{3*3*4*4*10}\right)=\log\frac{7}{10}=\log\frac{x}{10}$

   x = 7; Option C

5. $X = 25^{10}$

   LogX = 10Log25 = 20Log5 = 20(1 − Log2) = 20(1 − 0.30103) = 13.97

   Characteristics of LogX = 13

   Thus number of digits in X = C + 1 = 13 + 1 = 14; Option C

6. $9^{1/3}.9^{1/9}.9^{1/27}.9^{1/81}.$

   $= 9^{1/3+1/9+1/27+1/81} = 9^{40/81} = 3^{80/81}$

   Option D

7. $X = 2^{1/3} = 2^{6/18} = 64^{1/18}$

   $Y = 3^{1/6} = 3^{3/18} = 27^{1/18}$

   $Z = 4^{1/9} = 4^{2/18} = 16^{1/18}$

   X > Y > Z. Option B

8. AB = 36. Possible pairs (1, 36) (2, 18) (3, 12) (4, 9) (6, 6)

C = |A − B| = 35, 16, 9, 5, 0

Option C

9. $X = 3 + \sqrt{8}$

$X^{-1} = (3 - \sqrt{8})/(9-8) = 3 - \sqrt{8}$

$X + X^{-1} = 6$

$X^2 + X^{-2} = 36 - 2 = 34$

$X^4 + X^{-4} = 34^2 - 2 = 1156 - 2 = 1154$

Option A

10. $A^z = B^y = C^x = K$

$K^{2/y} = K^{1/z+1/x}$

2xz = y(x + z); Option B

11. $\log\left(\frac{a+b}{7}\right) = \frac{1}{2}(\log a + \log b)$

$2\log\left(\frac{a+b}{7}\right) = \log(ab)$

$(a+b)^2 = 49ab$

$a^2 + b^2 + 2ab = 49ab$; Option B

12. $X = \frac{1}{6}\cdot\sqrt{\frac{3\log(1728)}{1+\frac{1}{2}\log(0.36)+\frac{1}{3}\log 8}}$

$X = \frac{1}{6}\cdot\sqrt{\frac{3\log(1728)}{1+\log(0.6)+\log 2}} = \frac{1}{6}\cdot\sqrt{\frac{9\log 12}{\log 12}}$

$= \frac{1}{6}\sqrt{9} = \frac{1}{6} \times 3 = \frac{1}{2}$

$\therefore \quad x = \frac{1}{2}$

$\therefore \quad 2x = 1$

$\therefore \quad 2x - 1 = 0$

Option C

13. $x=\frac{6ab}{a+b};\frac{x}{3a}=\frac{2b}{a+b};\frac{x+3a}{x-3a}=\frac{a+3b}{b-a}$

$x=\frac{6ab}{a+b};\frac{x}{3b}=\frac{2a}{a+b};\frac{x+3b}{x-3b}=\frac{3a+b}{a-b}$

$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}=\frac{-a-3b+3a+b}{(a-b)}=\frac{2a-2b}{a-b}=2$

Option C

14. $3\sqrt{2}+7\sqrt{8}+\sqrt{27}+5\sqrt{3}=3\sqrt{2}+14\sqrt{2}+3\sqrt{3}+5\sqrt{3}=17\sqrt{2}+8\sqrt{3}$

Option B

15. $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$

$\frac{2\sqrt{x+4}}{2\sqrt{x-10}}=\frac{5+2}{5-2}=\frac{7}{3}$

$\frac{x+4}{x-10}=\frac{49}{9};\frac{2x-6}{14}=\frac{58}{40};\frac{x-3}{7}=\frac{29}{20}$

20x − 60 = 203; x = 263/20. Option C

16. a : b = 3 : -1 find $\frac{ab}{a+b}$

a & b $=\frac{ab}{(a+b)}$, find 3 & (& -1)

3 & -1 $=\frac{3(-1)}{3-1}=\frac{-3}{2}$

Then, 3 & (3 &- 1) $=\frac{(3)\left(\frac{-3}{2}\right)}{3-\left(\frac{3}{2}\right)}=\frac{\frac{-9}{2}}{\frac{3}{2}}$ = -3

17. $\log\frac{12}{13}-\log\frac{7}{25}+\log\frac{91}{3}=\log\left(\frac{2*2*3}{13}x\frac{5*5}{7}x\frac{13*7}{3}\right)=\log100$

x = log100 = 2. Option C

18. $\frac{1}{1+\frac{1}{x}}=\frac{x}{x+1}=x(x+1)^{-1}$

Option C

19. $\frac{a^3-b^3}{a^3+b^3}=\frac{13}{14};\frac{2a^3}{-2b^3}=\frac{27}{-1};\frac{a^3}{b^3}=\frac{27}{1};\frac{a}{b}=\frac{3}{1};\frac{a+b}{a-b}=\frac{4}{2}=2$

Option B

20. $x + x^{-1} = 3$

$x^2 + x^{-2} = 9 - 2 = 7$

$x^3 + x^{-3} = (3)(7-1) = 18$

$x^6 + x^{-6} = 18^2 - 2 = 324 - 2 = 322$. Option D

21. $2 + \sqrt{3} = 2 + 1.732 = 3.732$

$\sqrt{5} + \sqrt{2} = 2.23 + 1.41 = 3.64$

Option C

22. $\dfrac{(y-1)(y-2)(y^2-9y+14)}{(y-7)(y^2-3y+2)} = \dfrac{(y-1)(y-2)(y-2)(y-7)}{(y-7)(y-2)(y-1)} = (y-2)$

Option C

23. $(x + y) = 8/2 = 4$; $xy = 1$

$$\frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{(x+y)^2-xy}{(x+y)^2-3xy} = \frac{16-1}{16-3} = \frac{15}{13}$$

Option B

24. $$\frac{1}{\sqrt{11-2\sqrt{30}}} - \frac{3}{\sqrt{7-2\sqrt{10}}} - \frac{4}{\sqrt{8+4\sqrt{3}}}$$

$$= \frac{1}{\sqrt{6}-\sqrt{5}} - \frac{3}{\sqrt{5}-\sqrt{2}} - \frac{4}{\sqrt{6}+\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{6-5} - \frac{3(\sqrt{5}+\sqrt{2}}{(5-2)} - \frac{4(\sqrt{6}-\sqrt{2}}{(6-2)}$$

$$= \sqrt{6}+\sqrt{5}-\sqrt{5}-\sqrt{2}-\sqrt{6}+\sqrt{2} = 0$$

Option D

25. $X = \text{Log}(5*5*5*16/2) = \text{Log}(1000) = 3$

$(X + 3) = 3 + 3 = 6$. Option B

26. $\text{Log}X = \text{Log}(1.5*12) = \text{Log}18$.

$X = 18$. $X/3 = 18/3 = 6$. Option B

27. $(X - 13).8 = (3X + 1)$

$8X - 104 = 3x + 1$; $5X = 105$; $X = 21$. Option A

28. $\log\left(\frac{81}{100}\right)^2 * \log\left(\frac{27}{10}\right)^{2/3} \div \log 9$

$= (8\log 3 - 4)(2\log 3 - 2/3) * (1/2\log 3)$

= (3.8168 − 4) (0.9542 − 0.6667)(1/0.9542) = (- 0.1832)(0.2875)/(0.9542)

= - 0.0552

Option B

29. E= $\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = \frac{\log xy}{\log xyz} + \frac{\log yz}{\log xyz} + \frac{\log zx}{\log xyz} = \log_{xyz}(xyz)^2 = 2$

(E − 3) = (2 − 3) = - 1

Option C

30. $\log\frac{75}{35} + 2\log\frac{7}{5} - \log\frac{105}{x} - \log\frac{13}{25} = 0$

$\log\left(\frac{75*49*25}{35*25*13}\right) = \log\frac{105}{13} = \log\frac{105}{x}$

x = 13

Option A

2 EQUATIONS

PART A - THEORY

Equations

An equation is defined as a mathematical statement of equality.

Types of Equations

a) Linear equation in one variable.
b) Linear simultaneous equations in 2 or 3 variables.
c) Quadratic equations.
d) Cubic equations.
e) Bi-quadratic equations.
f) Exponential equations.

Quadratic Equations

- A quadratic equation is defined as polynomial equation of degree 2.
- A quadratic equation can be expressed in the following general form:

$$ax^2 + bx + c = 0;\ (a \neq 0)$$

- A quadratic equation can also be expressed in the factor form as follows:

$$a(x-\alpha)(x-\beta) = 0$$

Here, α and β are the roots or solutions of quadratic equations.

- The general solution of the quadratic equation can be obtained as follows:

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a}$$

Structure of Quadratic Equations

If Sum (S) (α + β) and Product (P) (αβ) of the roots are known, then the quadratic equation is

$$x^2 - Sx + P = 0$$

Sign of Roots of a Quadratic Equation

- When c=0, one root of the equation must be 0.
- When b and c are 0, then both the roots must be 0.
- If a, b, c all are of same sign, both roots are negative.
- If a and c are of same sign, opposite to that of b, then both the roots will be positive.
- If a and c are of opposite signs, one root is positive and another root is negative.

Nature of Roots

The expression "$b^2 - 4ac$" is called the "Discriminant (D)" of the quadratic equation.

- When $D > 0$, Roots are real and distinct.
- When $D = 0$, Roots are real and equal.
- When $D < 0$, Roots are imaginary.
- When $D \geq 0$, Roots are real.
- When D is a perfect square, Roots are real, rational and unequal.
- When D is not a perfect square, Roots are real, irrational and unequal.
- If roots are equal use $b^2 = 4ac$.
- If roots are reciprocal of each other, use $a = c$
- If roots are equal but of opposite sign, use $b = 0$
- If roots are reciprocal but opposite in sign, use $c = -a$

Note

- Irrational roots will always appear in conjugate pairs.

$$\alpha = \left(a - \sqrt{b}\right) \text{ and } \beta = \left(a + \sqrt{b}\right)$$

- Imaginary roots will always appear in conjugate pairs

$$\alpha = (a - ib) \text{ and } \beta = (a + ib)$$

Cubic Equations

- A cubic equation is a polynomial equation of degree 3, and the general form is represented as follows:

$$ax^3 + bx^2 + cx + d = 0;\ (a \neq 0)$$

- The factor form of a cubic equation is given as follows:

$$a(x - \alpha)(x - \beta)(x - \gamma) = 0$$

Here, α, β, and γ are the roots or solutions of the cubic equation.

- Sum of roots = $\alpha + \beta + \gamma = -b/a$
- Product of the roots = $\alpha\beta\gamma = -d/a$

Bi-Quadratic Equations

- A bi-quadratic equation is a polynomial of degree 4, and the general form is represented as follows:

$$ax^4 + bx^3 + cx^2 + dx + e = 0;\ (a \neq 0)$$

- The factor form of a cubic equation is given as follows:

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$$

Here, α, β, γ and δ are the roots or solutions of the bi-quadratic equation.

- Sum of roots = $\alpha + \beta + \gamma + \delta = -b/a$
- Product of the roots = $\alpha\beta\gamma\delta = e/a$

PART B - CLASSWORK

Choose the most appropriate option (a), (b), (c) or (d).

1. The sum of two numbers is 52 and their difference is 2. The numbers are

   a) 17 and 15 b) 12 and 10

   c) 27 and 25 d) none of these

2. The diagonal of a rectangle is 5 cm and one of its sides is 4 cm. Its area is

   a) 20 sq.cm. b) 12 sq.cm.

   c) 10 sq.cm. d) none of these

3. Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are.

   a) (20, 36) b) (25, 31)

   c) (24, 32) d) none of these

4. The sum of the digits of a two digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is

   a) 37 b) 73

   c) 75 d) none of these numbers.

5. The fourth part of a number exceeds the sixth part by 4. The number is

   a) 84 b) 44

   c) 48 d) none of these

6. Ten years ago, the age of a father was four times of his son. Ten years hence, the age of the father will be twice that of his son. The present ages of the father and the son are.

   a) (50, 20) b) (60, 20)

   c) (55, 25) d) none of these

7. The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2. The numbers are

   a) (16, 200) b) (160, 20) c) (60, 30) d) (80, 40)

8. One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer then the number was

a) 320 b) 400 c) 480 d) none of these.

9. If a number of which the half is greater than 1/5 th of the number by 15 then the number is

a) 50 b) 40 c) 80 d) none of these.

10. Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves ₹ 50 per month find their monthly incomes.

a) (500, 400) b) (400, 500) c) (300, 600) d) (350, 550)

11. The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

a) 60 years b) 52 years c) 51 years d) 50 years

12. A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.

a) 54 b) 53 c) 45 d) 55

13. The wages of 8 men and 6 boys amount to ₹ 33. If 4 men earn ₹ 4.50 more than 5 boys determine the wages of each man and boy.

a) (₹ 1.50, ₹ 3) b) (₹ 3, ₹ 1.50) c) (₹ 2.50, ₹ 2) d) (₹ 2, ₹ 2.50)

14. y is older than x by 7 years 15 years back, x's age was 3/4 of y's age. Their present ages are:

a) (x=36, y=43) b) (x=50, y=43)

c) (x=43, y=50) d) (x=40, y=47)

15. The sum of the digits in a three digit number is 12. If the digits are reversed the number is increased by 495 but reversing only of the ten's and unit digits increases the number by 36.The number is

a) 327 b) 372 c) 237 d) 273

16. The demand and supply equations for a certain commodity are $4q + 7p = 17$ and $p = \frac{q}{3} + \frac{7}{4}$. respectively where p is the market price and q is the quantity then the equilibrium price and quantity are:

(a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None of these.

17. The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are

a) (7, 10) b) (4, 4) c) (3, 5) d) (2, 6)

18. Five times of a positive whole number is 3 less than twice the square of the number. The number is

a) 3 b) 4 c) −3 d) 2

19. Two squares have sides p cm and (p + 5) cms. The sum of their squares is 625 sq. cm. The sides of the squares are

a) (10 cm, 30 cm) b) (12 cm, 25 cm)
c) 15 cm, 20 cm) d) none of these

20. Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are

a) (24, 26) b) (28, 22) c) (27, 23) d) (20, 30)

21. There are two consecutive numbers such that the difference of their reciprocals is 1/240.The numbers are

a) (15, 16) b) (17, 18) c) (13, 14) d) (12, 13)

22. The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are

a) (15, 30) b) (32, 13) c) (36, 9) d) (25, 20)

23. The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right angle triangle is formed. The side of the equilateral triangle is

a) 17 units b) 16 units c) 15 units d) 18 units

PAST YEAR QUESTIONS

24. Area and perimeter of rectangle is 6000 cm^2 and 340 cm length of rectangle is
(a) 140 (b) 120 (c) 170 (d) 200

25. If length of rectangle is 5 cm more than the breadth and if perimeter of rectangle is 40 cm, length and breadth of rectangle will be
(a) 7.5 cm, 2.5 cm (b) 10 cm, 5 cm
(c) 12.5 cm, 7.5 cm (d) 15.5 cm, 10.5 cm

26. Number of students in each section of a school is 36. After admitting 12 new students, four new sections are started. If total number of students in each section now is 30, then number of section initially were
(a) 6 (b) 10 (c) 14 (d) 18

QUADRATIC EQUATIONS

1. Equation : $x^2 + x + 1 = 0$ roots are

   (a) Real and equal (b) Real and unequal

   (c) Imaginary (d) Real and rational

2. For what value of 'c', the roots of the equation $2x^2 - 10x + c = 0$ are real and equal

   (a) 25/2 (b) 25/4 (c) 25/3 (d) none

3. If '−4' is a root of the equation $x^2 + ax - 4 = 0$ and the equation $x^2 + ax + b = 0$ has equal roots, the value of 'a' & 'b' are

   (a) $a = 2, b = \frac{5}{4}$ (b) $a = 3, b = \frac{9}{4}$

   (c) $a = , b = \frac{5}{2}$ (d) none

4. If the roots of equation $x^2 + (2k - 1)x + k^2 = 0$ are real, condition is

   (a) $k \geq 1$ (b) $k \leq 4$

   (c) $k \geq \frac{1}{4}$ (d) $k \leq \frac{1}{4}$

5. If the equation $x^2 - (b + 4)x + 2b + 5 = 0$ has equal roots, then the values of 'b'

   (a) −2 (b) 2 (c) ±2 (d) ±1

6. If α and β are roots of $x^2 + 2x + 1 = 0$, $\alpha^3 + \beta^3 =$

   (a) 2 (b) −2 (c) 4 (d) −4

7. If $p + q + r = 0$ and p, q, r are rational nos. the roots of equation $(q + r - p)x^2 + (r + p - q)x + (p + q - r) = 0$

   (a) real and irrational (b) real & equal

   (c) imaginary (d) real & rational

8. If one root of the equation $x^2 - 8x + k = 0$ exceeds the other by 4, value of k is

   (a) k = 10 (b) k = 11 (c) k = 9 (d) k = 12

9. If one root is double the other for the equation $ax^2 + bx + c = 0$, then

   (a) $b^2 = 4ac$ (b) $2b^2 = 9ac$ (c) $3b^2 = 10ac$ (d) $4b^2 = 9ac$

10. If roots of the equation $ax^2 + bx + c = 0$ are in the ratio $\frac{1}{m}$, then value of b^2/ac is

(a) $\frac{(l+m)^2}{lm}$ (b) $\frac{l+m}{lm}$ (c) $\left(\frac{l-m}{lm}\right)^2$ (d) $\frac{l-m}{lm}$

11. If α, β are roots of equation $x^2 - 5x + 6 = 0$, $\alpha > \beta$, then equation with roots $\alpha + \beta$, $\alpha - \beta$ as

(a) $x^2 - 6x + 5 = 0$ (b) $2x^2 - 6x + 5 = 0$

(c) $2x^2 - 5x + 6 = 0$ (d) $x^2 - 5x + 6 = 0$

12. The values of $4 + \cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\ldots\infty}}}$

(a) $1 \pm \sqrt{2}$ (b) $2 + \sqrt{5}$ (c) $2 \pm \sqrt{5}$ (d) None of these

13. If the sum of the roots of the quadratyic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to

a) 2 b) −2 c) 1 d) −1

14. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as $\frac{p}{q}$ and $\frac{q}{p}$ is

(a) $x^2 - 19x + 3 = 0$ (b) $3x^2 - 19x - 3 = 0$

(c) $3x^2 - 19x + 3 = 0$ (d) $3x^2 + 19x + 3 = 0$

15. If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other then the value of p is

a) −5 b) 5 c) 1/5 d) −1/5

PAST YEARS QUESTIONS

1. On solving $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, we get one value of x as:

(a) $\frac{4}{13}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{3}{13}$

2. Find the positive value of k for which the equation : $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots

(a) 12 (b) 16 (c) 18 (d) 22

3. If one root of an equation is $3 + \sqrt{5}$, then the quadratic equation is

(a) $x^2 - 6x + 4 = 0$ (b) $x^2 + 6x - 4 = 0$

(c) $x^2 + 6x + 4 = 0$ (d) $x^2 - 6x - 4 = 0$

4. If $(2 + \sqrt{3})$ is a root of a quadratic equation $x^2 + px + q = 0$ then find value of p and q

(a) (4, −1) (b) (4, 1) (c) (−4, 1) (d) (2, 3)

5. One root of the equation $x^2 - 2(5 + m)x + 3(7 + m) = 0$ is reciprocal of the other. Find the value of m

(a) $\frac{-64}{7}$ (b) 7 (c) 1/7 (d) $\frac{-20}{3}$

6. If one root of the equation $x^2 - 3x + k = 0$ is 2 then value of k will be

(a) −10 (b) 0 (c) 2 (d) 10

7. If roots of equation $x^2 + x + r = 0$ are α and β and $\alpha^3 + \beta^3 = -6$. Find value of r

(a) $-\frac{5}{3}$ (b) $\frac{7}{3}$ (c) $-\frac{4}{3}$ (d) 1

8. If one root of the equation $px^2 + qx + r = 0$ is r then other root of the equation will be

(a) 1/q (b) 1/r (c) 1/p (d) $\frac{1}{p+q}$

9. If the ratio of the root of the equation $4x^2 - 6x + p = 0$ is 1 : 2 then the value of p is

(a) 1 (b) 2 (c) −2 (d) −1

10. If p and q are the root of equation $x^2 - bx + c = 0$ then what is the equation whose roots are (pq + p + q) and (pq − p − q)

(a) $x^2 - 2cx + c^2 - b^2 = 0$ (b) $x^2 - 2cx + c^2 + b^2 = 0$

(c) $cx^2 - 2(a + c)x + c^2 = 0$ (d) $x^2 + 2bx - (c^2 - b^2) = 0$

11. If α, β are the roots of the quadratic equation $2x^2 - 4x = 1$ then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

(a) −11 (b) 22 (c) −22 (d) 11

12. If difference between the roots of the equation $x^2 - kx + 8 = 0$ is 4 then the value of k is

(a) 0 (b) ±4 (c) $\pm 8\sqrt{3}$ (d) $\pm 4\sqrt{3}$

CUBIC EQUATION

Choose the most appropriate option (a), (b), (c) or (d)

1. The solution of the cubic equation $x^3-6x^2+11x-6=0$ is given by the triplet:

   a) -1, 1 -2 b) 1, 2, 3 c) -2, 2, 3 d) 0, 4, -5

2. The cubic equation $x^3+2x^2-x-2=0$ has 3 roots namely.

   a) 1, -1, 2 b) -1, 1, -2 c) -1, 2, -2 d) 1, 2, 2

3. x,x-4,x+5 are the factors of the left-hand side of the equation.

   a) $x^3 + 2x^2-x-2=0$ b) $x^3+x^2-20x=0$

   c) $x^3-3x^2-4x+12=0$ d) $x^3-6x^2+11x-6=0$

4. The equation $3x^3+ 5x^2=3x+5$ has got 3 roots and hence the factors of the left-hand side of the equation $3x^3+5x^2-3x-5=0$ are

   a) x-1,x-2,x-5/3 b) x-1,x+1, 3x +5

   c) x+1,x-1,3x-5 d) x-1,x+1,x-2

5. The roots of the equation $x^3+7x^2-21x-27=0$ are

   a) (-3,-9,-1) b) (3,-9,-1) d) (3,9,1) e) (-3,9,1)

6. If $4x^3+8x^2-x-2=0$ than value of (2x+3)is given by

   a) 4, -1, 2 b) -4, 2, 1 c) 2, -4, -1 d) None of these

7. The rational root of the equation $2x^3-x^2-4x+2=0$ is

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2

BI QUADRATIC EQUATION

1. Solving equation $6x^4 + 11x^3 - 9x^2 -11x + 6 = 0$ following roots are obtained

   a) $\frac{1}{2}, -2, \frac{-1\pm\sqrt{37}}{6}$ b) $-\frac{1}{2}, 2, \frac{-1\pm\sqrt{37}}{6}$

   c) $\frac{1}{2}, -2, \frac{5}{6}, \frac{-7}{6}$ d) None

2. Find the roots of the equation: $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

a) 1, 2, ½ b) – 1, 2, ½ c) – 2, 1, ½ d) – ½, 1, 2

CONSISTENCY OF EQUATION

1. The system of equation $5x - 4y = 7$ and $3x - 2y = 15$ have

(a) unique solution (b) infinite solution

(c) no solution (d) none

2. The system of equation $9x - 17y = 34$ and $36x - 68y = 115$ have

(a) unique Solution (b) infinite Solution

(c) no solution (d) none

3. The system of equation $6x + 5y = 11$ and $9x + (15/2)y = 21$ have

(a) unique Solution (b) infinite Solution

(c) no solution (d) none

4. The system of equation $4x + 7y = 10$ and $10x + (35/2)y = 25$ have

(a) unique solution (b) infinite solution

(c) no solution (d) none

5. The value of k for which the system of equations: $7x - y = 5$; $21x - 3y = k$ have infinite solution is:

(a) k = 4 (b) k = 15 (c) k =3 (d) k=7

6. Determine the values of a and b for which the following system of linear equations has consistent infinite many solutions: $2x - (a-4)y = 2b + 1$ and $4x - (a-1)y = 5b - 1$

(a) a = -7, b =3 (b) a = 7, b =3

(c) a = -7, b = -3 (d) None

EQUATION HOMEWORK

1. The solution of the equation $(p+2)(p-3)+(p+3)(p-4)= p(2p-5)$ is

   (a) 6 (b) 7 (c) 5 (d) none of these

2. The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for

   (a) x=1 (b) x = 2 (c) x = 5 (d) x = 7

3. Solve for x and y : $\frac{4}{x} - \frac{5}{y} = \frac{x+y}{xy} + \frac{3}{10}$ and $3xy= 10(y-x)$

   (a) (5,2) (b) (-2,-5) (c) (2,-5) (d) (2,5)

4. The simultaneous equations $7x-3y = 31$, $9x-5y = 41$ have solutions given by

   (a) (−4, −1) (b) (−1, 4) (c) (4, −1) (d) (3, 7)

5. $\frac{xy}{x+y} = 20$, $\frac{yz}{y+z} = 40$, $\frac{zx}{z+x} = 24$

   (a) (120, 60, 30) (b) (60, 30, 120)
   (c) (30, 120, 60) (d) (30, 60, 120)

6. $\frac{xy}{y-x} = 110$, $\frac{yz}{z-y} = 132$, $\frac{zx}{z+x} = 60/11$

   (a) (12, 11, 10) (b) (10, 11, 12)
   (c) (11, 10, 12) (d) (12, 10, 11)

7. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal then value of m is

   (a) − 3 (b) − 1 (c) 1 (d) − 2

8. If $2^{2x+3} - 3^2 . 2^x + 1 = 0$ then values of x are

   (a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, − 3

9. A solution of the quadratic equation $(a+b-2c)x^2 + (2a-b-c)x + (c+a-2b) = 0$ is

   a) x = 1 b) x = −1 c) x = 2 d) x = − 2

10. If the root of the equation $x^2-8x+m = 0$ exceeds the other by 4 then the value of m is

    a) m = 10 b) m = 11 c) m = 9 d) m = 12

11. The values of x in the equation
$7(x+2p)^2 + 5p^2 = 35xp + 117p^2$ are
(a) (4p, −3p) (b) (4p, 3p) (c)(−4p, 3p) (d) (−4p, −3p)

12. The solutions of the equation $\frac{6x}{x+1} + \frac{6(x+1)}{x} = 13$ are
(a) (2, 3) (b) (3, −2) (c) (−2, −3) (d) (2, −3)

13. The satisfying values of x for the equation $\frac{1}{x+p+q} = \frac{1}{x} + \frac{1}{p} + \frac{1}{q}$ are
a) (p, q) b) (−p, −q) c) (p, −p) d) (−p, q)

14. The values of x for the equation $x^2 + 9x + 18 = 6 - 4x$ are
a) (1, 12) b) (−1, −12) c) (1, −12) d) (−1, 12)

15. The values of x satisfying the equation
$\sqrt{(2x^2+5x-2)} - \sqrt{(2x^2+5x-9)} = 1$ are
(a) (2, −9/2) (b) (4, −9) (c) (2, 9/2) (d) (−2, 9/2)

16. The solution of the equation $3x^2-17x + 24 = 0$ are
(a) (2,3) (b) $\left(2, 2\frac{2}{3}\right)$ (c) $\left(3, 2\frac{2}{3}\right)$ (d) $\left(3, \frac{2}{3}\right)$

17. The equation $\frac{3(3x^2+15)}{6} + 2x^2 + 9 = \frac{2x^2+96}{7} + 6$ has got the solution as
a) (1, 3) b) (1/2, −1) c) (1, −1) d) (2, −1)

18. Solving equation $x^2 - (a+b)x + ab = 0$ are, value(s) of x
(a) a, b (b) a (c) b (d) None

19. Solving equation $x^2 - 24x + 135 = 0$ are, value(s) of x
(a) 9, 6 (b) 9, 15 (c) 15, 6 (d) None

20. If $\frac{x}{b} + \frac{b}{x} = \frac{a}{b} + \frac{b}{a}$ the roots of the equation are
(a) a, b^2 / a (b) a^2 , b/a^2 (c) a^2, b^2/a (d) a, b^2

21. Solving equation $3x^2-14x+16 = 0$ we get roots as
(a) ± 1 (b) 2 and $\frac{8}{3}$ (c) 0 (d) None

22. Solving equation $3x^2 - 14x + 8 = 0$ we get roots as

(a) ±4 (b) ± 2 (c) $4, \frac{2}{3}$ (d) None

23. Solving equation $(b-c)\,x^2 + (c-a)\,x + (a-b) = 0$ following roots are obtained

(a) $\frac{a-b}{b-c}, 1$ (b) $(a-b)(c-a), 1$ (c) $\frac{b-c}{a-b}, 1$ (d) None

24. Solving equation $7\sqrt{\frac{x}{1-x}} + 8\sqrt{\frac{1-x}{x}} = 15$ following roots are obtained

(a) $\frac{64}{113}, \frac{1}{2}$ (b) $\frac{1}{50}, \frac{1}{65}$ (c) $\frac{49}{50}, \frac{1}{65}$ (d) $\frac{1}{50}, \frac{64}{65}$

25. If α, β are the roots of equation $x^2-5x+6=0$ and $\alpha > \beta$ then the equation with roots $(\alpha\beta + \alpha + \beta)$ and $(\alpha\beta - \alpha - \beta)$ is

(a) $x^2 - 12x + 11 = 0$ (b) $2x^2 - 6x + 12 = 0$

(c) $x^2 - 12x + 12 = 0$ (d) None

26. Solving $x^3+9x^2 - x - 9 = 0$ we get the following roots

(a) ±1, -9 (b) ±1, ±9 (c) ±1, 9 (d) None

27. Solve $x^3 - 7x^2 + 14x - 8 = 0$ given that the roots are in geometrical progression

(a) ½, 1, 2 (b) 1, 2, 4 (c) ½ , -1, 2 (d) -1, 2, -4

28. Solve $x^3 - 6x^2 + 5x + 12 = 0$ given that the product of the two roots is 12

(a) 1, 3, 4 (b) -1, 3, 4 (c) 1, 6, 2 (d) 1, -6, -2

HOMEWORK SOLUTIONS

1. The solution of the equation $(p + 2)(p - 3) + (p + 3)(p - 4) = p(2p - 5) = ?$

$(p + 2)(p - 3) + (p + 3)(p - 4) = p(2p - 5)$

$\Rightarrow (p^2 + 2p - 3p - 6) + (p^2 + 3p - 4p - 12) = 2p^2 - 5p$

$\Rightarrow 2p^2 - 2p - 18 = 2p^2 - 5p$

$\Rightarrow 5p - 2p = 18$

$\Rightarrow 3p = 18 \Rightarrow p = 6$

2. The equation $\frac{12x + 1}{4} = \frac{15x - 1}{5} + \frac{2x - 5}{3x - 1}$ is true for?

$\Rightarrow \frac{12x}{4} + \frac{1}{4} = \frac{15x}{5} - \frac{1}{5} + \frac{2x - 5}{3x - 1}$

$\Rightarrow 3x + \frac{1}{4} = 3x - \frac{1}{5} + \frac{2x - 5}{3x - 1}$

$\Rightarrow \frac{1}{4} + \frac{1}{5} = \frac{2x - 5}{3x - 1}$

$\Rightarrow \frac{9}{20} = \frac{2x - 5}{3x - 1}$

Now, if $x = 7$, $\frac{9}{20} = \frac{2(7) - 5}{3(7) - 1} = \frac{14 - 5}{21 - 1} = \frac{9}{20}$

3. (d)

4. (c)

5. $\frac{xy}{x + y} = 20$; $\frac{yz}{y + z} = 40$; $\frac{zx}{z + x} = 24$

Here, $(x > y > z)$ $\therefore$ We pick option (d) 30, 60, 120

also check, $\frac{(30)(60)}{30 + 60} = \frac{180}{90} = 20$

(Ascending order) $\frac{(60)(120)}{60+120} = \frac{7200}{180} = 40$

6. $\frac{xy}{y-x} = 110$; $\frac{yz}{z-y} = 132$; $\frac{zx}{z+x} = \frac{60}{11}$ $x < y < z$

(Ascending order) $\therefore$ 10, 11, 12

also, $\frac{(10)(11)}{11-10} = 110$; $\frac{(11)(12)}{12-11} = 132$

7. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal, then value of m is:

When the roots are equal $b^2 = 4ac$

Here, $a = 2$; $b = 8$; $c = -m^3$

$\therefore 64 = (4)(2)(-m^3)$

$\Rightarrow \frac{64}{8} = -m^3$

$\therefore m = -2$

$\Rightarrow -8 = -m^3$

$m = -2$

8. If $2^{2x+3} - 3^2 . \times + 1 = 0$; x = ?

M–I

$2^{2x} . 2^3 - 3^2 . 2^x + 1 = 0$

$\Rightarrow 8x^{2x} - 9 . 2^x + 1 = 0$

$\Rightarrow 8t^2 - 9t + 1 = 0$

$\Rightarrow 8t^2 - 8t^2 - 1t + 1 = 0$

$\Rightarrow 8t(t-1) - 1(t-1) = 0$

$\Rightarrow t = \frac{1}{8}$; $t = 1$

Let $2^x = t$

Now, $2^x = \frac{1}{8} = \frac{1}{2^3}$

$2^x = 2^{-3}$

$\therefore x = -3$

$2^x = 1 = 2^\circ$

$\therefore x = 0$

M–II

Plug in option (d) (0, –3)

Put $x = 0$, $2^{0+3} - 3^2 \cdot 2^0 + 1$

$= 8 - 9 + 1 = 0$

$x = -3 \quad 2^{-6+3} - 3^2 \cdot 2^{-3} + 1$

$\Rightarrow \frac{1}{8} - \frac{9}{8} + 1$

$-1 + 1 = 0$

9. (b)

10. (d) $\alpha + \beta = 8, \alpha - \beta = 4$

$2\alpha = 12, \alpha = 6, \beta = 2$

$m = \alpha\beta = 12$

11. (a)

12. (d)

13. (b)

14. (b) Equation $x^2 + 13x + 12 = 0$

$\alpha\beta = 12, \alpha + \beta = -13$

15. (a)

16. (c) $\alpha\beta = 8, \alpha + \beta = 17$

17. (c)

18. (a) $\alpha + \beta = a + b, \alpha\beta = ab$

19. (b) $\alpha + \beta = 24, \alpha\beta = 135$

20. (a)

21. (b) $\alpha + \beta = \frac{14}{3}, \alpha\beta = \frac{16}{3},$

22. (c) $\alpha + \beta = \frac{14}{3}, \alpha\beta = \frac{8}{3}$

23. (a) Since, '1' is the roots in all 3 options

Let $\beta = 1$. We know that, $\alpha\beta = \frac{c}{a}$

$(\alpha)(1) = \frac{a-b}{b-c}$ $\therefore$ the other toot

$\alpha = \frac{a-b}{b-c}$

24. (a) $7\sqrt{\frac{x}{1-x}} + 8\sqrt{\frac{1-x}{x}} = 15$

M–I:

Let, $\sqrt{\frac{x}{1-x}} = k$, then $\sqrt{\frac{1-x}{x}} = \frac{1}{k}$

$\therefore\ 7k + \frac{8}{k} = 15 \Rightarrow$'s $7k^2 + 8 = 15k$

$7k^2 - 15k + 8 = 0$

$7k^2 - 7k - 8k + 8 = 0$

$7k(k-1) - 8(k-1) = 0$

$k = 1\ ;\ k = \frac{8}{7}$

Now, $\sqrt{\frac{x}{1-x}} = 1\ ;$ $\sqrt{\frac{x}{1-x}} = \frac{8}{7}$

$\frac{x}{1-x} = 1\ ;$ $\frac{x}{1-x} = \frac{64}{49}$

$x = 1 - x$ $49x = 64 - 64x$

$2x = 1\ \ x = \frac{1}{2}$ $113x = 64$

$\frac{64}{113} = x$

25. (a) If α, β are the roots of $x^2 - 5x + 6 = 0$, $\alpha > \beta$ then equation with $(\alpha\beta + \alpha + \beta)$ and $(\alpha\beta - \alpha - \beta) = ?$

$x^2 - 5x + 6 = 0$, $x = 2$ and $x = 3$ [on factorization]

$\therefore \alpha = 3$ and $\beta = 2$ $[\because \alpha > \beta]$

Now, $(\alpha\beta + \alpha + \beta) = [3 \times 2 + 3 + 2] = [6 + 5] = 11$

and $(\alpha\beta - \alpha - \beta) = [3 \times 2 - 3 - 2] = [6 - 5] = 1$

$\therefore$ the equation $(x^2 - 5x + p)$

$\Rightarrow x^2 - 12x + 11 = 0$

26. (a) Solving $x^3 + 9x^2 - x - 9 = 0$ we get; the foll roots.

Sol. We know that, if $\frac{a}{b} = \frac{c}{d}$, in $ax^3 + bx^2 + cx + d = 0$ then, we factorise,

$x^3 + 9x^2 - x - 9 = 0$

$\Rightarrow x^2(x + 9) - 1(x + 9) = 0$

$\Rightarrow (x^2 - 1)(x + 9) = 0$

$\Rightarrow x = \pm 1$; $x = -9$

27. Solve $x^3 - 7x^2 + 14x - 8 = 0$, roots are in G.P.

Sol. Among the options, (b) has 1, 2, 4. a, b, c

$\sqrt{ac} = b \quad \therefore 2 = \sqrt{1 \times 4}$

28. Solve $x^3 - 6x^2 + 5x + 12 = 0$, given the product of 2 roots is 12.

We know that, $\alpha\beta\gamma = \frac{-d}{a} = \frac{-12}{1}$

Here, a = 1 ; b = −6 ; c = 5 ; d = 12

Only option (b) satisfies with −1, 3, 4 as $-1 \times 3 \times 4 = -12$

Again, chk, sum of the roots,

$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-6)}{1} = 6$

4 + 3 − 1 = 7 − 1 = 6.

SELF ASSESSMENT TEST 3
Equations 1
13 Question, 13 Marks

1. If sum of three numbers is 25, sum of product of numbers in pairs is 250, what is the sum of square of numbers?
   a) 250 b) 125 c) 375 d) 300

2. Find the value of $(1.348)^3 + 3(1.348)(1.304) + (0.652)^3$
   a) 2 b) 8 c) 2.258 d) 8.258

3. Solve for X and Y: 1.5X + 2.4Y = 1.8 and 2.5(X + 1) = 7Y
   a) 0.5, 0.5 b) 0.4, 0.4 c) 0.4, 0.5 d) 0.5, 0.4

4. If 3 chairs and 2 tables cost Rs. 12000 and 5 chairs and 3 tables cost Rs. 19000, then the cost of 2 chairs and 2 tables is:
   a) Rs. 9000 b) Rs. 7000 c) Rs. 10000 d) Rs. 11000

5. Find the value of K for which the system of equations 2x + 2y = 5 and 3x + Ky = 7 has no solution.
   a) 9 b) 5 c) 7 d) 3

6. Find the value of k for which the system of equations 2x + ky = 1; 3x − y = 7 has a unique solution.
   a) k = - 2/3 b) $\neq \frac{2}{3}$
   c) $k \neq -\frac{2}{3}$ d) None of the above

7. Find the value of $(1.729)^3 + 3(1.729)(0.542) + (0.271)^3$
   a) 2 b) 8 c) 2.271 d) 1.458

8. A man has some hens and cows. If the number of heads be 48 and number of feet equals 140, the number of hens will be:
   a) 22 b) 23 c) 24 d) 26

9. Ramesh bought a horse for Rs. X. He sold it at 0.9X, thereby registering 10% loss. Had the horse been sold at Rs. 4500 more Ramesh would have make a profit of 12.5%. Find X.

a) Rs. 20,000 b) Rs. 18,000 c) Rs. 200,000 d) Rs. 150,000

10. Given $x \in \{-3, -4, -5, -6\}$ and $9 \leq 1 - 2x$, find the possible values of x.

a) $\{-3, -4, -5, -6\}$ b) $\{-4, -5, -6\}$

c) $\{-3, -5, -6\}$ d) None of the above

11. Give the solution set for $3 - 2x \geq x - 32$, given that $x \in N$.

a) $\{1, 2, 3, 4, 5\}$ b) $\{-3, -2, 1, 2, 3, 4, 5, 6, 7\}$

c) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ d) None of the above

12. Dipti needs a minimum of 360 marks in four tests in a Statistics course to obtain an A grade. On her first three tests, she scored 88, 96, 79 marks. What should her score (X) be in the fourth test so that she can make an A grade?

a) $X > 95$ b) $X \geq 95$ c) $X \geq 97$ d) None of the above

13. Which of the following is the solution set for $|x + 2| \geq 5$?

a) $\{x : x \in R, x < -7 \text{ or } x \geq 3\}$

b) $\{x : x \in R, x \leq -7 \text{ or } x \geq 3\}$

c) $\{x : x \in R, x \leq -7 \text{ or } x > 3\}$

d) None of the above

EXPLANATORY ANSWERS

1. $(A + B + C) = 25$, $(AB + BC + CA) = 250$
   $(A^2 + B^2 + C^2) = (A + B + C)^2 - 2(AB + BC + CA) = 625 - 500 = 125$; Option B

2. $(1.348)^3 + 3(1.348)(1.304) + (0.652)^3$
   $= (1.348)^3 + (0.652)^3 + 3(1.348)(0.652)(1.348 + 0.652)$
   $= (1.348 + 0.652)^3 = 2^3 = 8$; Option B

3. Option C (Using options).

4. $3C + 2T = 12000$; $5C + 3T = 19000$
   $2C + T = 7000$
   $C + T = 5000$
   $2C + 2T = 10000$. Option C

5. $2/3 = 2/K$. $K = 3$. Option D

6. $2/3 \neq k/-1$, Option C

7. $(1.729)^3 + 3(1.729)(0.542) + (0.271)^3$
   $= (1.729)^3 + (0.271)^3 + 3(1.729)(0.271)(1.729 + 0.271)$
   $= (1.729 + 0.271)^3 = 23 = 8$
   Option B

8. $H + C = 48$; $2H + 4C = 140$; $H = 26$. Option D

9. $CP = X$
   New $SP = 0.9X + 4500$
   Profit $= 0.9X + 4500 - X = 4500 - 0.1X$
   $(4500 - 0.1X)/X = 0.125$
   $4500 - 0.1X = 0.125X$
   $0.225X = 4500$
   $X = 4500/0.225 = 20000$; Option A

10. $9 \leq 1 - 2x$

$x \leq -4$

$x = \{-4, -5, -6\}$

Option B

11. $3 - 2x \geq x - 32$

$35 \geq 3x$

$x \leq 11.67$

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Option D

12. $88 + 96 + 79 = 263$

$x \geq (360 - 263) = 97$

Option C

13. $(x + 2) \leq -5; x \leq -7$

$(x + 2) \geq 5; x \geq 3$

Option B

SELF ASSESSMENT TEST 4
Equations 2
15 Question, 15 Marks

1. If A and B are the roots of the equation $16x^2 - 8x + 1 = 0$, then which of the following is true?
   a) A, B are real
   b) A, B are real and $A \neq B$
   c) A, B are real and $A = B$
   d) A and B, both are imaginary

2. If M and N are the roots of the equation $2(A^2 + B^2)X^2 + 2(A + B)x + 1 = 0$, then which of the following is true?
   a) M, N are real
   b) M, N are Imaginary
   c) M, N are Distinct
   d) Both b) & c) above

3. If one root of the equation $4x^2 + 5x + K = 0$ be reciprocal of another root, but negative in sign, then what can be the value of K?
   a) 4
   b) – 4
   c) 2.75
   d) – 3.25

4. If the roots of the equation $1/(x + 2) + 1/(x + 3) = 2/5$, are equal in magnitude but opposite in sign, then the product of the roots is:
   a) 1
   b) 2.5
   c) –13
   d) – 6.5

5. 'A' meters of cloth costs $35. If this piece of cloth would had been 4 m longer and each meter costs $ 1 less, the cost of the cloth would had been $35. What can be the value of A?
   a) 8
   b) 10
   c) 12
   d) 14

6. Find the maximum value of the expression $x^2 - 4x + 7$, for real value of x.
   a) 3
   b) 8
   c) 9
   d) Undeterminable

7. For the given bi-quadratic equation: $4x^4 - 16x^3 + 7x^2 + 16x + 4 = 0$, what is the value of product of all possible roots of the equation?
   a) – 4
   b) 4
   c) – 16
   d) 1

8. For the given equation: $(x^2 + 2)^2 + 8x^2 - 6x(x^2 + 2) = 0$, what is the sum of the roots of the equation?

a) 8 b) 6 c) − 6 d) 0

9. If one root of the equation $14x^2 + 5^3x + K = 0$ be reciprocal of another root, find the value of K.

a) 7 b) 14 c) 14/125 d) None of the above

10. For the given equation: $(x^2 + 2)^2 + 8x^2 - 6x(x^2 + 2) = 0$, what is the product of the roots of the equation?

a) − 4 b) 4 c) 3 d) None of the above

11. Which of the following equation in variable X have two roots, 2 and 4?

a) $X^2 - 6X - 8 = 0$ b) $X^2 + 6X - 8 = 0$

c) $X^2 + 6X + 8 = 0$ d) $X^2 - 6X + 8 = 0$

12. Two roots of a quadratic equation $x^2 + x - 6 = 0$ are A and B respectively. If A > B, then which of the following quadratic equation will have roots − A and B?

a) $x^2 - 5x + 6 = 0$ b) $x^2 + 5x - 6 = 0$

c) $x^2 + 5x + 6 = 0$ d) $x^2 - 5x - 6 = 0$

13. Which of the following is one of the factor of the equation $x^4 - 19x^2 + 6x + 72 = 0$?

a) (x + 3) b) (x − 2) c) (x − 4) d) (x − 3)

14. Which of the following cubic equation have factors (x − 2), (2x + 3) and (x − 3)?

a) $2x^3 - 7x^2 - 9x + 18 = 0$ b) $x^3 - 7x^2 - 9x + 18 = 0$

c) $x^3 - 7x^2 + 9x + 18 = 0$ d) $2x^3 + 7x^2 - 9x + 18 = 0$

15. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq b)$ have a common root, then:

a) a + b = ab b) $a + b = a^2 + b^2$ c) $a^2 + b^2 = ab$ d) a + b + 1 = 0

EXPLANATORY ANSWERS

1. Discriminant (D) = 64 – 4*16*1 = 0. Roots are Real and Equal. Option C

2. $D = 4(A + B)^2 - 4*2(A^2 + B^2)*1 = 4[A^2 + B^2 + 2AB - 2A^2 - 2B^2] = -4(A - B)^2 < 0$
   Thus M and N are imaginary and distinct. Option D

3. Product of roots = K/4 = -1. K = - 4. Option B

4. $5(x + 2 + x + 3) = 2(x + 2)(x + 3)$
   $10x + 25 = 2x^2 + 10x + 12$
   2x2 – 13 = 0
   Product of roots = -13/2 = - 6.5
   Option D

5. Cost per meter = 35/A
   $(A + 4)(35/A - 1) = 35$
   $35 - A + 140/A - 4 = 35$
   $A^2 + 4A - 140 = 0$
   A = 10
   Option B

6. $x^2 - 4x + 7$
   $= x^2 - 2*2*x + 2^2 + 3$
   $= (x - 2)^2 + 3$
   The expression is positive for any value of x> = 2
   Maximum value is undeterminable. Option D

7. Product of roots = +e/a = 4/4 = 1. Option D

8. $x^4 + 4 + 4x^2 + 8x^2 - 6x^3 - 12x = 0$
   $x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$
   Sum of roots = -b/a = -(-6)/1 = 6
   Option B

9. Product = 1 = K/14. K = 14. Option B

10. $(x^2 + 2)^2 + 8x^2 - 6x(x^2 + 2) = 0$

$x^4 + 4x^2 + 4 + 8x^2 - 6x^3 - 12x = 0$

$x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

Product of roots = 4/1 = 4

Option B

11. $(X - 2)(X - 4) = 0$

$X^2 - 6X + 8 = 0$; Option D

12. $x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$

A = 2, B = - 3

B = - 3 and - A = - 2

Required equation is $(x + 3)(x + 2) = 0$

$x^2 + 5x + 6 = 0$

Option C

13. $F(x) = x^4 - 19x^2 + 6x + 72$

$F(3) = 3^4 - 19(3)^2 + 6*3 + 72 = 81 - 171 + 18 + 72 = 0$

$(x - 3)$ is one of the factor. Option D

14. $(x - 2)(2x + 3)(x - 3) = 0$

$(2x^2 - x - 6)(x - 3) = 0$

$2x^3 - 7x^2 - 9x + 18 = 0$

Option A

15. $k^2/(a^2 - b^2) = k/(b - a) = 1/(b - a)$

$k = (a + b)$ or 1

$1 + a + b = 0$

Option D

3 TIME VALUE OF MONEY

Simple Interest

Simple interest is charged on the principal amount and hence it is same for every year.

A = Amount, P = principal, n = number of years, R = interest rate

$$SI = \frac{PTR}{100}$$

$$A = P + SI = P + \frac{PTR}{100} = P\left(1 + \frac{TR}{100}\right)$$

Notes:

- If rate of interest is known, then sum of money will double itself in 100/r years.
- If number of years is known, then sum of money will double itself @ 100/n %.
- A sum of money will become "n" times in $\frac{(n-1) \times 100}{R}$ years.

Example:

In how many years a sum of money @10% p.a. SI will become (a) double, (b) triple, (c) N times.

(a) Double	(b) Triple	(c) N times
$\frac{(2-1) \times 100}{10} = 10$ years	$\frac{(3-1) \times 100}{10} = 20$ years	$\frac{(N-1) \times 100}{10} = 10(N-1)$ years

- If the sum of money becomes "n_1" times in T_1 years and "n_2" times in T_2 years, then the ratio of their times is: $\frac{T_1}{T_2} = \frac{n_1 - 1}{n_2 - 1}$.

Compound Interest

- In case of compound interest, the interest is calculated on the amount of the succeeding years, i.e., principal keeps changing every year.

- Here interest on interest is also earned, thus money grow faster when Compounding is done

- If P is the principal, n = number of years for which interest is calculated and "i" (R/100) is the rate of interest, then, the amount A after n years will be given by:

$$A=P(1+i)^n$$

- In case of depreciation by diminishing balance method (WDV), if C = Cost of the machinery, I = rate of depreciation per annum and n = effective life of the machinery, then the depreciated value D after n years is :

$$D = C(1 - i)^n$$

D is also known as the scrap value of the machinery.

- Compound Interest thus would be calculated as follows:

$$CI = A - P = P\left[(1+i)^n - 1\right]$$

- Depending upon the compounding style of interest rate, the effective formula for calculating Amount would be as follows:

Half Yearly or Semi Annually	Quarterly	Monthly
$A = P\left(1+\frac{i}{2}\right)^{2n}$	$A = P\left(1+\frac{i}{4}\right)^{4n}$	$A = P\left(1+\frac{i}{12}\right)^{12n}$

- When differential interest rates are charged (i_1, i_2, i_3,... ... i_n), then:

$$A = P(1+i_1)(1+i_2)(1+i_3).....(1+i_n)$$

- Relationship between CI and SI
  - a) For the first year, CI = SI, i.e. for the first year difference is zero.
  - b) For two years, CI − SI = Pi^2
  - c) For three years, CI − SI = $Pi^2(i + 3)$

Notes:

1. A sum of money will double itself in approximately 72/r years (known as Rule 72), where r is the rate of interest per annum.

2. A sum of money will triple itself in approximately 114/r (known as Rule 114), where r is the rate of interest per annum.

3. If a sum of money becomes "n" times in "t" years, then, it will become n^m times in "mt" years.
   Example: If sum of money doubles itself in 3 years, then it will be 8 times (2^3) in 3x3 = 9 years at CI.

Concept of Effective Rate of Interest

1. When the compounding is done more than once a year, then, the net annual rate of interest is found to be slightly higher than the given annual rate of interest.

2. This new rate of interest is known as the effective rate of interest and the given annual rate is called the nominal rate of interest.

3. Effective rate of interest is denoted by E and is given by the formula:

$$E = \left\{ (1+i)^n - 1 \right\} \text{ x } 100$$

Where "i" is rate of interest, converted monthly, quarterly, half yearly and n is the number of conversion period per annum.

4. Effective rate of interest are particularly useful in making investment decisions when various options are given with differential interest rates.

5. Amongst various investment options, we shall choose that investment option, where effective rate of interest is maximum.

Concept of Present Value

Present Value is defined as the present worth of the money that would yield an amount A after n years at a specified rate of interest i.

$$\text{If } A = P(1+i)^n$$

$$\therefore P = PV = \text{Principal} = \frac{A}{(1+i)^n}$$

$$or, PV = A(1+i)^{-n}$$

Annuities

- Annuity is defined as a series of payments (usually equal) which are made at regular intervals of time (usually a year).

- The period for which the payment continues is called the status or the term of the annuity.

- Unless otherwise stated, the first payment will fall due at the end of every year. This is known as "Ordinary Annuity".

- When the payment falls due at the beginning of every year, i.e., immediately, it is called "Immediate Annuity".

- When the status or term of the annuity is not fixed, i.e., the payment is to be continued for an indefinite period, these are known as "Perpetual Annuity or Perpetuity".

- Hence forth, we shall maintain the following notation throughout. The regular annual payment i.e., annuity = P, rate of interest = "i" and the period for which payment is made = n (status or term of the annuity).

- The amount of the ordinary annuity is given by:

$$A = \frac{P}{i}\left\{(1+i)^n - 1\right\}$$

- The amount of immediate annuity is obtained by multiplying amount obtained for ordinary annuity by (1 + i); hence the formula becomes:

$$A = \frac{P}{i}\left\{(1+i)^n - 1\right\}(1+i)$$

- Note:
  1. When half yearly or quarterly or monthly payment is "P", in such a case change "i" to i/2 or i/4 or i/12 and change "n" to 2n or 4n or 12n respectively.

  2. When half yearly, quarterly or monthly rate of interest is "i", in such a case, change P to P/2, P/4 or P/12 and change n to 2n or 4n or 12n respectively.

- The present value of an annuity payable over a period of n years is defined as the sum of the present value of all the future payments.

- The present value of an ordinary annuity is represented by V and given as follows:

$$V = \frac{P}{i}\left\{1-(1+i)^{-n}\right\}$$

- If the term of the annuity is n years, then for evaluating the present value of the immediate annuity, first calculate the present value of the annuity for (n−1) years and then add to it the initial or first payment.

$$v = \frac{P}{i}\left\{1-(1+i)^{-n}\right\}(1+i)$$

- Present value of the perpetual annuity is given by,

$$V = P/i$$

CLASSWORK SECTION

SIMPLE INTEREST

1. How much interest will be earned on Rs. 4000 at 6% p.a. simple interest for 2 yrs?
   (a) 450 (b) 480 (c) 500 (d) 540

2. A deposited 1,00,000 in a bank for 2 years with the interest at 5.5% p.a. What will be the final value of investment?
   (a) 1,00,000 (b) 1,11,000 (c) 1,20,000 (d) 1,30,000

3. Find rate of interest if the amount owed after 6 months is 2100, borrowed amount being Rs. 2000.
   (a) 10% (b) 8% (c) 9% d 11%

4. P = 5000, N = 1, I = 300, R will be
   (a) 5% (b) 4% (c) 6% (d) none

5. 46875 was lent out at SI and at the end of 1 yr 8 months, total amount was 50000. Find rate of int per annum?
   (a) 2% (b) 4% (c) 6% (d) 8%

6. Sum required to earn quarterly interest of 3600 at 18% p.a. is
   (a) 50,000 (b) 60,000 (c) 80,000 (d) none

7. In how much time would SI on a certain sum be 0.125 times the principal at 10% p.a.?
   (a) 1 ¼ years (b) 1.5 years (c) 1 ¾ years (d) 2 ¼ years

8. A sum of 3402 amounts to 6804 on 20 yrs. What sum will amount to 5200 in 6 yrs at same rate?
   (a) 3000 (b) 4000 (c) 5000 (d) 600

9. 30000 is invested in two parts : partly at 10% p.a. and partly at 15% p.a. Total interest earned is 3300. How much is invested at lower rate?
   (a) 20000 (b) 24000 (c) 26000 (d) 28000

10. A bike is purchased by making a down payment of 15000 and balance to be paid alongwith interest at 5% p.a. for 2 yrs. Total amount paid is 28200. Find cash price of the bike.

(a) 28000 (b) 26000 (c) 27000 (d) 25000

11. A certain sum amounts to 7400 in 3 yrs and 8600 in 4 yrs. Find the sum and rate of interest

(a) 3800, 31.57% (b) 3,000, 25%

(c) 3,500, 20% (d) none

12. A certain sum doubles itself in 20 yrs. In how many years it will become 7 times

(a) 100 (b) 120 (c) 140 (d) none

13. Mr. X takes loan of 7000 for 8 yrs. After 3 yrs he taken 3000 more. Total interest paid at the end of 8 years is 3550. Find the rate of interest.

(a) 4% p.a. (b) 5% p.a. (c) 6% p.a. (d) none

PAST EXAM QUESTIONS

14. ₹ 8,000 becomes ₹ 10,000 in two years at simple interest. The amount that will become ₹ 6,875 in 3 years at the same rate of interest is:

(a) ₹ 4,850 (b) ₹ 5,000 (c) ₹ 5,500 (d) ₹ 5,275

15. The rate of simple interest on a sum of money is 6% p.a. for first 3 years, 8% p.a. for the next five years and 10% p.a. for the period beyond 8 years. If the simple interest accrued by the sum for a period for 10 years is ₹ 1,560. The sum is:

(a) ₹ 1,500 (b) ₹ 2,000 (c) ₹ 3,000 (d) ₹ 5,000

16. A sum of money doubles itself in 10 years. The number of years it would treble itself is :

(a) 25 years (b) 15 years (c) 20 years (d) none

17. If ₹ 1,000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to ₹ 2,000 is:

(a) $16\frac{2}{3}$ years (b) $16\frac{1}{4}$ years

(c) 16 years (d) $16\frac{1}{3}$ years

18. Two equal sums of money were lent at simple interest at 11% p.a. for $3\frac{1}{2}$ years and $4\frac{1}{2}$ years respectively. If the difference in interests for two periods was ₹ 412.50, then each sum is:

(a) ₹ 3,250 (b) ₹ 3,500 (c) ₹ 3,750 (d) ₹ 4,350

19. Find the numbers of years in which a sum doubles itself at the rate of 8% per annum.

(a) $11\frac{1}{2}$ (b) $12\frac{1}{2}$

(c) $9\frac{1}{2}$ (d) $13\frac{1}{2}$

20. If a simple interest on a sum of money at 6% p.a. for 7 years is equal to twice of simple interest on another sum for 9 years at 5% p.a. The ratio will be

(a) 2 : 15 (b) 7 : 15 (c) 15 : 7 (d) 1 : 7

21. If the simple interest on ₹ 1,400 for 3 years is less than the simple interest on ₹ 1,800 for the same period by ₹ 80, then the rate of interest is

(a) 5.67% (b) 6.67% (c) 7.20% (d) 5.00%

22. The S.I. on a sum of money is $\frac{4}{9}$ of the principal and the no. of years is equal to the rate of interest per annum. Find the rate of interest per annum?

(a) 5% (b) 20/3% (c) 22/7% (d) 6%

23. Mr. X invests ₹ 90,500 in post office at 7.5% p.a. simple interest. While calculating the rate was wrongly taken as 5.7% p.a. The difference in amounts at maturity is ₹9,774. Find the period for which the sum was invested:

(a) 7 years (b) 5.8 years (c) 6 years (d) 8 years

24. A sum of ₹ 44,000 is divided into three parts such that the corresponding interest earned after 2 years, 3 years and 6 years may be equal. If the rates of simple interest are 6% p.a., 8% p.a. and 6% p.a. respectively, then the smallest part of the sum will be :

(a) ₹ 4,000 (b) ₹ 8,000 (c) ₹ 10,000 (d) ₹ 12,000

25. A person borrows ₹ 5,000 for 2 years at 4% per annual simple interest. He immediately lends to another person at $6\frac{1}{4}$ %. Per annual for 2 years find his gain in the transaction for per year:

(a) ₹ 112.50 (b) ₹ 225 (c) ₹ 125 (d) ₹ 107.50

26. A man invests an amount of ₹ 15,860 in the names of his three sons A, B and C in such a way that they get the same SI after 2, 3 and 4 years respectively. If the rate of interest is 5%, then the ratio of amount invested in the name of A, B and C is

(a) 6 : 4 : 3 (b) 3 : 4 : 6 (c) 30 : 12 : 5 (d) none of these

COMPOUND INTEREST

27. Find amount for a sum of 4000 at 8% p.a. for 5 yrs compounded annually?

(a) 5877 (b) 6577 (c) 8577 (d) 5677

28. Find C–I for a sum of 8000 at 4% p.a. for 6 yrs compounded half yearly?

(a) 2146 (b) 2416 (c) 2164 (d) 2641

29. Find amount and C–I for a sum of 6000 at 12% p.a. for 3 years compounded quarterly?

(a) 8554.5, 2554.5 (b) 7554.5,1554.5
(c) 9554.5, 3554.5 (d) 6554.5,554.5

30. Find amount for a sum of 10000 at 6% p.a. for 2 years compounded monthly?

(a) 12171 (b) 11712 (c) 11271 (d) 12117

31. Find present value of 10000 due in 2 yrs at 5% p.a. compound interest paid annually?

(a) 9050 (b) 9070 (c) 9080 (d) 9090

32. Find present value of 20000 due in 3 yrs at 6% p.a. C–I paid half yearly?
(a) 16570 (b) 16500 (c) 16750 (d) 16075

33. A machinery is depreciated at 10% p.a. for 3 yrs costing Rs. 50000.Find scrap value?
(a) 36400 (b) 36450 (c) 36500 (d) 36550

34. Find depreciation if machinery worth 12000 is depreciated at 6% p.a. for 4 yrs?
(a) 2631.8 (b) 2613.8 (c) 2361.8 (d) 2316.8

35. A machinery worth 10000 is depreciated at the rate of 10% p.a. for first 3 yrs. 8% p.a. for next 2 yrs. Find its value after 5 yrs.
(a) 5170.25 (b) 7170.25 (c) 6170.25 (d) 8170.25

36. An investment of 20000 sums on interest of 6% p.a. for first 4 yrs 5% p.a. for next 3 yrs and 4% p.a. for next 2 yrs. Find value of investment after 9 yrs where interest is compound annually
(a) 31441.62 (b) 31614.62 (c) 31416.62 (d) 31641.62

37. Difference between C–I and SI at 5% p.a. for 2 yrs on a sum of 6000 is
(a) 10 (b) 12 (c) 15 (d) 18

38. Difference between CI and SI on 10000 at 5% p.a. for 4 yrs is
(a) 150 (b) 155 (c) 160 (d) 165

39. A sum of money at 5% p.a. CI double in
(a) 14 yrs (approx.) (b) 16 yrs (approx.)
(c) 18 yrs (approx.) (d) 19 yrs (approx.)

40. In how many years a sum of money trebles at 5% p.a. CI payable on half yearly basis
(a) 20 yrs 3 months (b) 21 yrs 3 months
(c) 22 yrs 3 months (d) 24 yrs 3 months

41. A sum at a certain rate of interest compounded annually doubles in 5 yrs. In how many yrs will it become 8 times
(a) 10 (b) 15 (c) 18 (d) 20

PAST EXAM QUESTIONS

42. The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is

(a) ₹ 3,175 (b) ₹ 3,075 (c) ₹ 3,275 (d) ₹ 2,975

43. In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% per annum, when the interest is compounded semi-annually?

(a) 2 years (b) 4 years (c) 5 years (d) 7 years

44. The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is:

(a) 35 years (b) 30 years (c) 25 years (d) none of these

45. A person deposited ₹ 5000 in a bank. The deposit was left to accumulate at 6% compounded quarterly for the first five years and at 8% compounded semi-annually for the next eight years. The compound amount at the end of 13 years is:

(a) ₹ 12621.50 (b) ₹ 12613.10 (c) ₹ 13613.10 (d) none

46. A sum amount to ₹ 1,331 at a principal of ₹ 1,000 at 10% compounded annually. Find the time.

(a) 3.31 years (b) 4 years (c) 3 years (d) 2 years

47. Mr. X invests 'P' amount at simple interest rate 10% and Mr. Y invests 'Q' amount at compound interest rate 5% compounded annually. At the end of two years both get the same amount of interest, then the relation between two amounts P and Q is given by

(a) $P = \frac{41Q}{80}$ (b) $P = \frac{41Q}{40}$

(c) $P = \frac{41Q}{100}$ (d) $P = \frac{41Q}{100}$

48. A sum of money compounded annually becomes ₹ 1,140 in two years and ₹ 1,710 in three years. Find the rate of interest per annum.

(a) 30% (b) 40% (c) 50% (d) 60%

49. A sum of money invested of compound interest doubles itself in four years. It becomes 32 times of itself at the same rate of compound interest in

(a) 12 years (b) 16 years (c) 20 years (d) 24 years

50. A compound interest on a sum for 2 years is ₹ 30 more than the simple interest at the rate of 5% per annum then the sum is

(a) ₹ 11,000 (b) ₹ 13,000 (c) ₹ 12,000 (d) ₹ 15,000

51. If compound interest on any sum at the rate of 5% for two years is ₹ 512.50 then the sum would be:

(a) ₹ 3,000 (b) ₹ 4,000 (c) ₹ 5,000 (d) ₹ 6,000

52. If compound interest on a sum for 2 years at 4% per annum is ₹ 102, then the simple interest on the same sum for the same period at the same rate will be

(a) ₹ 99 (b) ₹ 101 (c) ₹ 100 (d) ₹ 95

EFFECTIVE RATE OF INTEREST

53. Effective annual rate of interest compounding to a nominal rate of 6% p.a payable half yearly is

(a) 6.07 (b) 6.08 (c) 6.09 (d) none

54. Effective rate of interest of 8% p.a. converted monthly is

(a) 8% (b) 8.34% (c) 8.43% (d) 8.30%

55. Which is a better investment?

(i) 9% p.a. compounded half yrly.

(ii) 9.23% p.a. S.I.

(a) (i) (b) (ii) (c) both (d) none

ANNUITY (FUTURE VALUE)

56. The amount of annuity of 6,000 payable at the end of each 3 months for 4 years compounded Quarterly at 8% p.a.

(a) 111836 (b) 110836 (c) 112836 (d) 113836

57. The amount of annuity of Rs. 2000 payable at the end of each year for 5 years at 8% p.a. is

(a) 11733.86 (b) 14502.6 (c) 21005.8 (d) 16721.31

58. A company requires 20,00,000 at the end of 10 yrs to replace one of its assets. It is decided to create a sinking fund by investing a fixed amount every year in securities which gives 10% CI. Yearly investment is

(a) 124590 (b) 125490 (c) 154290 (d) 145290

59. A company issued 10% cumulative debentures of Rs. 100 each, 5000 cumulative debentures are to be redeemed with 10% of interest for 5 yrs. For this a Sinking Fund is created and invested at 12% rate of C.I. Sum to be transferred every year to sinking fund is

(a) 805500 (b) 126834.64 (c) 207382 (d) 126755

60. The value of the amount at the end of 12 years of an annuity of 1200 payable at the beginning of each year for 12 yrs at 8% p.a. C.I is

(a) 26879.32 (b) 3432.11 (c) 24594.35 (d) none

61. A machine costing 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by new model at 25% higher cost after 25 years with a scrap value realization of 25000. What amount should be set aside every year if sinking fund investment at 3.5% C.I p.a.?

(a) 16000 (b) 16500 (c) 16050 (d) 16005

62. Ratan aged 45 wishes his wife Ratna to have 40 lacs at his death. His expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% p.a., how much should he invest annually?

(a) 84077 (b) 81628 (c) 84450 (d) none

ANNUITY (PRESENT VALUE)

63. A loan of 30,000 at the interest rate of 6% compounded annually is to be amortized by equal payments at the end of each year for 5 years. Find annual payment.

(a) 6121.89 (b) 7121.89 (c) 8121.89 (d) 9121.89

64. Present value of an annuity which pays 200 at the end of each 3 months for 10 years, assuming money to be worth 5% p.a. converted quarterly.
(a) 3809.20 (b) 3109.60 (c) 6265.38 (d) none

65. Pravin buys a house paying Rs. 50,000 in cash and balance in 20 installments of Rs. 8,000 each at the end of each year. If interest is at 16% p.a., how much he should have paid if had purchased it cash down.
(a) 85250 (b) 94730 (c) 97430 (d) 87589

66. P.V. of an annuity of Rs. 80 made at the end of each 6 months forever, if money worth 4% p.a. is compounded semi-annually
(a) 2000 (b) 3000 (c) 4000 (d) 5000

67. A man purchased house valued at 3,00,000 by making a payment of 2,00,000 at the time of purchase and agreed to pay balance with interest at 12% p.a. compounded half yearly in 20 equal half yearly installments. If first installment is paid after 6 months from the date of purchase then amount of each installment is
(a) 8719 (b) 8679 (c) 7719 (d) 8769

68. John wants to create a fund to donate 1800 every month to a deprived family. Rate of unit is 12% p.a. Find amount to be deposited
(a) 360000 (b) 180000 (c) 90000 (d) none

69. A company borrows 10000 on condition to repay it with C.I. at 5% p.a. by annual installments of 1000 each. The number of years by which debt will be cleared is
(a) 14.2 (b) 10 (c) 12 (d) 17

70. A person retires at 60 years receiving a pension of 14,400 a year paid in half yearly installments for the rest of his life with his life expectation to be 13 years and interest at 4% p.a. payable half yearly. What single sum is equivalent to his pension?
(a) 144000 (b) 144900 (c) 144600 (d) 144300

71. If discount rate is 7% p.a., how much would you pay to receive 500, growing at 5% annually forever?
(a) 25000 (b) 250000 (c) 2500 (d) none

PAST YEARS QUESTION

72. A machine can be purchased for ₹50,000. Machine will contribute ₹12,000 per year for the next five years. Assume borrowing cost is 10% per annum. Determine whether machine should be purchased or not:

(a) Should be purchased (b) Should not be purchased

(c) Can't say about purchase (d) None of the above

73. A company considering proposal of purchasing a machine either by, making full payment of ₹4000 or by leasing it for four years at an annual rate of ₹1,250. Which course of action is preferable, if the company can borrow money at 14% compounded annually?

[Given: $(1.14)^4 = 1.68896$]

(a) Leasing is preferable (b) Should be Purchased

(c) No difference (d) None of these

74. Vipul Purchases a car for ₹5,50,000. He gets a loan of ₹5,00,000 at 15% P.a. from a bank and balance ₹50,000 he pays at the time of purchase. He has to pay the whole amount of loan in 12 equal monthly instalments with interest starting from the end of the first month. The money he has to pay at the end of every month is:

[Given $(1.0125)^{12} = 1.16075452$]

(a) ₹ 45,130.43 (b) ₹ 45,230.43 (c) ₹ 45,330.43 (d) ₹ 45,430.43

75. A company establishes a sinking fund to provide for the payment of ₹ 2,00,000 debt maturing in 20 years. Contributions to the fund are to be made at the end of every year. Find the amount of each annual deposit if Interest is 5% per annum.

(a) ₹ 6,142 (b) ₹ 6,049 (c) ₹ 6,052 (d) ₹ 6,159

76. A company may obtain a machine either by leasing it for 5 years (useful life) at an annual rent of ₹2,000 or by purchasing the machine for ₹8,100. If the company can borrow money at 18% per annum, which alternative is preferable?

(a) Leasing (b) Purchasing (c) Can't say (d) None of these

77. A sinking fund is created for redeeming debentures worth ₹5 lacs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a?

(a) ₹ 12,006 (b) ₹ 12,040 (c) ₹ 12,039 (d) ₹ 12,035

78. Find the present value of an annuity of ₹1,000 payable at the end of each year for 10 years. If rate of interest is 6% compounding per annum (Given $(1.06)^{-10} = 0.5584$):

(a) ₹ 7,360 (b) ₹ 8,360 (c) ₹ 12,000 (d) None of these

79. The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually. [Given $(1.09)^8 = 1.99256$] is _____

(a) ₹ 55,142.22 (b) ₹ 65,142.22

(c) ₹ 65,532.22 (d) ₹ 57,425.22

80. A person wants to lease out a machine costing ₹ 5,00,000 for a 10 year period. It has fixed a rental of ₹ 51, 272 per annum payable annually starting from the end of first year. Suppose rate of interest is 10% per annum compounded annually on which money can be invested. To whom this agreement is favourable?

(a) Favour of lessee (b) Favour of lessor

(c) Not for both (d) Can't be determined

HOMEWORK SECTION

1. S.I on ₹ 3,500 for 3 years at 12% per annum is
   (a) ₹ 1,200 (b) ₹ 1,260 (c) ₹ 2,260 (d) none of these

2. P = 5,000, R = 15, T = 4 ½ using I = PRT/100, I will be
   (a) ₹ 3,375 (b) ₹ 3,300 (c) ₹ 3,735 (d) none of these

3. If P = 5,000, T = 1, I = ₹ 300, R will be
   (a) 5% (b) 4% (c) 6% (d) none of these

4. If P = ₹ 4,500, A = ₹ 7,200, than Simple interest i.e. I will be
   (a) ₹ 2,000 (b) ₹ 3,000 (c) ₹ 2,500 (d) ₹ 2,700

5. P = ₹ 12,000, A = ₹ 16,500, T = 2 ½ years. Rate percent per annum simple interest will be
   (a) 15% (b) 12% (c) 10% (d) none of these

6 P = ₹ 10,000, I = ₹ 2,500, R = 12 ½% SI. The number of years T will be
   (a) 1 ½ years (b) 2 years (c) 3 years (d) none of these

7. P = ₹ 8,500, A = ₹ 10,200, R = 12 ½ % SI, t will be.
   (a) 1 yr. 7 mth. (b) 2 yrs. (c) 1 ½ yr. (d) none of these

8. The sum required to earn a monthly interest of ₹ 1,200 at 18% per annum SI is
   (a) ₹ 50,000 (b) ₹ 60,000
   (c) ₹ 80,000 (d) none of these

9. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of simple interest are
   (a) ₹ 3,800, 31.57% (b) ₹ 3,000, 20%
   (c) ₹ 3,500, 15% (d) none of these

10. A sum of money doubles itself in 10 years at simple interest. The number of years it would triple itself is

(a) 25 years. (b) 15 years. (c) 20 years (d) none of these

11. If P = ₹ 1,000, R = 5% p.a, n = 4; What is Amount and C.I. is

(a) ₹ 1,215.50, ₹ 215.50 (b) ₹ 1,125, ₹ 125

(c) ₹ 2,115, ₹ 115 (d) none of these

12. ₹ 100 will become after 20 years at 5% p.a compound interest amount of

(a) ₹ 250 (b) ₹ 205 (c) ₹ 265.50 (d) none of these

13. The effective rate of interest corresponding to a nominal rate 3% p.a payable half yearly is

(a) 3.2% p.a (b) 3.25% p.a (c) 3.0225% p.a (d) none of these

14. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is

(a) 4.5 years (appx.) (b) 5.4 years (appx.)

(c) 5 years (appx.) (d) none of these

15. If A = ₹ 1,000, n = 2 years, R = 6% p.a compound interest payable half-yearly, then principal (P) is

(a) ₹ 888.48 (b) ₹ 885 (c) 800 (d) none of these

16. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

(a) 7 years (b) 10 years (c) 17 years (app) (d) none of these

17. The difference between C.I and S.I on a certain sum of money invested for 3 years at 6% p.a is ₹ 110.16. The sum is

(a) ₹ 3,000 (b) ₹ 3,700 (c) ₹ 12,000 (d) ₹ 10,000

18. The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciationis 10% p.a. The scrap value at the end of its life is

(a) ₹ 3,486.78 (b) ₹ 4,383 (c) ₹ 3,400 (d) none of these

19. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

(a) 7% (b) 7.5% (c) 5% (d) 7.18%

20. The C.I on ₹ 16000 for 1 ½ years at 10% p.a payable half -yearly is

(a) ₹ 2,222 (b) ₹ 2,522 (c) ₹ 2,500 (d) none of these

21. The C.I on ₹ 40000 at 10% p.a for 1 year when the interest is payable quarterly is

(a) ₹ 4,000 (b) ₹ 4,100 (c) ₹ 4,152.51 (d) none of these

22. The difference between the S.I and the C.I on ₹ 2,400 for 2 years at 5% p.a is

(a) ₹ 5 (b) ₹ 10 (c) ₹ 16 (d) ₹ 6

23. The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

(a) 35 years. (b) 30 years. (c) 25 years (d) none of these

24. The C.I on ₹ 4,000 for 6 months at 12% p.a payable quarterly is

(a) ₹ 243.60 (b) ₹ 240 (c) ₹ 243 (d) none of these

25. The present value of an annuity of ₹ 3000 for 15 years at 4.5% p.a CI is

(a) ₹ 23,809.41 (b) ₹ 32,218.63

(c) ₹ 32,908.41 (d) none of these

26. The amount of an annuity certain of ₹ 150 for 12 years at 3.5% p.a C.I is

(a) ₹ 2,190.28 (b) ₹ 1,290.28 (c) ₹ 2,180.28 (d) none of these

27. A loan of ₹ 10,000 is to be paid back in 30 equal instalments. The amount of each installment to cover the principal and at 4% p.a CI is

(a) ₹ 587.87 (b) ₹ 587 (c) ₹ 578.30 (d) none of these

28. A = ₹ 1,200 n = 12 years i = 0.08, V = ?

Using the formula $V = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$ value of v will be

(a) ₹ 3,039 (b) ₹ 3,990 (c) ₹ 9930 (d) none of these

29. p = ₹ 100 n = 10, i = 5% find the FV of annuity
(a) ₹ 1,258 (b) ₹ 2,581 (c) ₹ 1,528 (d) none of these

30. If the amount of an annuity after 25 years at 5% p.a C.I is ₹ 50,000 the annuity will be
(a) ₹ 1,406.90 (b) ₹ 1,047.62 (c) ₹ 1,146.90 (d) none of these

31. Given annuity of ₹ 100 amounts to ₹ 3137.12 at 4.5% p.a C. I. The number of years will be
(a) 25 years (appx.) (b) 20 years (appx.)
(c) 22 years (d) none of these

32. A company borrows ₹ 10,000 on condition to repay it with compound interest at 5% p.a by annual installments of ₹ 1000 each. The number of years by which the debt will be clear is
(a) 14.2 years (b) 10 years (c) 12 years (d) none of these

33. Mr. X borrowed ₹ 5,120 at 12 ½ % p.a C.I. At the end of 3 yrs, the money was repaid along with the interest accrued. The amount of interest paid by him is
(a) ₹ 2,100 (b) ₹ 2,170 (c) ₹ 2,000 (d) none of these

34. Mr. Paul borrows ₹ 20,000 on condition to repay it with C.I. at 5% p.a in annual installments of ₹ 2000 each. The number of years for the debt to be paid off is
(a) 10 years (b) 12 years (c) 11 years (d) none of these

35. A person invests ₹ 500 at the end of each year with a bank which pays interest at 10% p. a C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is.
(a) ₹ 11,761.36 (b) ₹ 10,000 (c) ₹ 12,000 (d) none of these

36. The present value of annuity of ₹ 5,000 per annum for 12 years at 4% p.a C.I. annually is
(a) ₹ 46,000 (b) ₹ 46,850 (c) ₹ 15,000 (d) none of these

37. A person desires to create a fund to be invested at 10% CI per annum to provide for a prize of ₹ 300 every year. Using V = a/I find V and V will be
(a) ₹ 2,000 (b) ₹ 2,500 (c) ₹ 3,000 (d) none of these

38. A = ₹ 5,200, R = 5% p.a., T = 6 years, P will be
(a) ₹ 2,000 (b) ₹ 3,880 (c) ₹ 3,000 (d) none of these

39. If P = 1,000, n = 4 years., R = 5% p.a then C. I will be
(a) ₹ 215.50 (b) ₹ 210 (c) ₹ 220 (d) none of these

40. The time in which a sum of money will be double at 5% p.a C.I is
(a) ₹ 10 years (b) 12 years (c) 14.2 years (d) none of these

41. If A = ₹ 10,000, n = 18yrs., R = 4% p.a C.I, P will be
(a) ₹ 4,000 (b) ₹ 4,900 (c) ₹ 4,500 (d) none of these

42. The time by which a sum of money would treble it self at 8% p. a C. I is
(a) 14.28 years (b) 14 years (c) 12 years (d) none of these

43. The present value of an annuity of ₹ 80 a years for 20 years at 5% p.a is
(a) ₹ 997 (appx.) (b) ₹ 900 (c) ₹ 1,000 (d) none of these

44. A person bought a house paying ₹ 20,000 cash down and ₹ 4,000 at the end of each year for 25 yrs. at 5% p.a. C.I. The cash down price is
(a) ₹ 75,000 (b) ₹ 76,000 (c) ₹ 76,392 (d) none of these.

45. A man purchased a house valued at ₹ 3,00,000. He paid ₹ 2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is
[Given log 10.6 = 1.0253 and log 31.19 = 1.494]
(a) ₹ 8,719.66 (b) ₹ 8,769.21 (c) ₹ 7,893.13 (d) none of these.

46. The difference between compound and simple interest at 5% per annum for 4 years on ₹ 20,000 is ₹ _____
(a) 250 (b) 277 (c) 300 (d) 310

47. The compound interest on half-yearly rests on ₹ 10,000 the rate for the first and second years being 6% and for the third year 9% p.a. is ₹._____.

(a) 2,200 (b) 2,287 (c) 2,285 (d) 2290

48. The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on yearly basis is ₹ _____.

(a) 9,070 (b) 9,000 (c) 9,061 (d) None

49. The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on half-yearly basis is ₹ _____.

(a) 9,070 (b) 9,069 (c) 9,059.50 (d) None

50. Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of simple interest being 3.5%, how much each son receive after getting 25 years old?

(a) 50,000 (b) 51,994 (c) 52,000 (d) 48332

51. In how many years will a sum of money double at 5% p.a. compound interest?

(a) 15 years 3 months (b) 14 years 2 months
(c) 14 years 3 months (d) 15 years 2 months

52. In how many years a sum of money trebles at 5% p.a. compound interest payable on halfyearly basis?

(a) 18 years 7 months (b) 18 years 6 months
(c) 18 years 8 months (d) 22 years 3 months

53. A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹ 23,240 and ₹ 9,000 respectively. For how many years the machine was put to use?

(a) 7 years (b) 8 years (c) 9 years (d) 10 years

54. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000?

(a) 4 years 6 months (b) 4 years 7 months
(c) 4 years 5 months (d) 5 years 7 months approximately

55. A machine worth ₹ 4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%?

(a) 11 years 6 months
(b) 11 years 7 months
(c) 11 years 8 months
(d) 14 years 2 months approximately

HOMEWORK SOLUTION

1. B

$I = \frac{3500 \times 3 \times 12}{100}$

$I = 1260$

2. A

$I = \frac{5000 \times 15 \times 4.5}{100} = 3375$

3. C

$300 = \frac{5000 \times 1 \times r}{100} = 6\%$

4. D

$I = A - P = 7200 - 4500$

$= 2700$

5. A

$I = A - P = 16,500 - 12,000$

$= 4,500$

$45,000 = \frac{12,000 \times R \times 2.5}{100} = 15\%$

6. B

$2500 \ \frac{10,000 \times 12.5 \times N}{100} = 2$ years

7. A

$I = 10,200 - 8,500 = 1700$

$1700 = \frac{85,00 \times 12.5 \times N}{100}$

$= 1.6$ years

$= (1.6 \times 12) \quad = 19.2$ month

$= 1$ year 7 months

8. C

$1200 = \frac{P(1)(18)}{100 \times 12} = 80,000$

9. A

After	3 years	=	7400
	2 years	=	6200
	1 year	=	1200

After 3 years	7400
(1200 x 3)	3600
Principle	3800

$12,00 = \frac{3800 \times 1 \times R}{100}$

$R = 31.57\%$

10. C

$R = \frac{(2-1)x\ 100}{10} = 10\%$

$N = \frac{(3-1)x\ 100}{10} = 20$ years

11. (a) P = 100, R = 5%, N = 4

$$A = P\left(1+\frac{R}{100}\right)^N$$

$$= 1000\left(1+\frac{5}{100}\right)^4$$

A = 1215.50

∴ CI = A – P

= 1215.50 – 1000

∴ CI = 215.50

12. (c) P = 100, R = 5%, r = 20 years

$$A = P\left(1+\frac{R}{100}\right)^N$$

$$= 100\left(1+\frac{5}{100}\right)^{20}$$

A = 265.50

13. (c) $E = [(1+i)^n - 1] \times 100$

$$= \left[\left(1+\frac{3}{200}\right)^2 - 1\right] \times 100$$

E = 3.0225%

14. (b) $A = P\left(1+\frac{R}{100}\right)^N$

$$\therefore 30{,}000 = 1{,}00{,}000\left(1-\frac{20}{100}\right)^N$$

$$\therefore 0.3 = (0.8)^N$$

∴ N = 5.4 years approx.

15. (a) $A = P\left(1+\frac{R}{100}\right)^N$

$$\therefore 1000 = P\left(1+\frac{6}{200}\right)^{2\times 2}$$

∴ P = 888.48

16. (c) $A = P\left(1+\frac{R}{100}\right)^N$

$$\therefore 140 = 100\left(1+\frac{2}{100}\right)^N$$

$$\therefore 1.40 = (1.02)^N$$

∴ N = 17 years approx

17. (d) $D = Pi^2(i+3)$

$$\therefore 110.16 = P(0.06)^2\,(0.06+3)$$

$$\therefore 110.16 = P(0.011016)$$

$$\therefore P = 9999.999 \approx 10{,}000.$$

18. (a) $A = P\left(1-\frac{R}{100}\right)^N$

$$\therefore A = 10{,}.000\left(1-\frac{10}{100}\right)^{10}$$

∴ A = 3486.78

19. (d) $E = [(1+i)^N - 1] \times 100$

$$= \left[\left(1+\frac{7}{400}\right)^4 - 1\right] \times 100$$

E = 7.18%

20. (b) $CI = P\left[\left(1+\frac{R}{100}\right)^N - 1\right]$

$$= 16{,}000\left[\left(1+\frac{10}{200}\right)^{1.5\times 2} - 1\right]$$

= 16,000 [0.157623]

CI = 2522.

21. (c) $CI = P\left[\left(1+\frac{R}{100}\right)^N - 1\right]$

$= 40{,}000\left[\left(1+\frac{10}{400}\right)^{1\times 4} - 1\right]$

$= 40{,}000\ [0.1038]$

$CI = 4152.51$

22. (d) $D = Pi^2$

$D = 2400\ (0.05)^2$

$D = 6$

23. (a) Rate of growth $= \frac{39.4 - 19.4}{1000} \times 100$

$= 2\ \%$

$A = P\left(1+\frac{R}{100}\right)^N$

$2 = 1\left(1+\frac{2}{100}\right)^N$

$\therefore$ N = 35 yrs. Approx.

OR

Approximate

No. of years $= \frac{72}{\text{Rate}} = 36 \approx 35$ years.

24. (a)

$CI = P\left[\left(1+\frac{R}{100}\right)^N - i\right]$

$= 4000\left[\left(1+\frac{12}{400}\right)^{\frac{1}{2}\times 4} - 1\right]$

$CI = 243.6$

25. (b) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{3000}{\frac{4.5}{100}}\left[1 - \frac{1}{\left(1+\frac{4.5}{100}\right)^{15}}\right]$

$= \frac{3000}{0.045}\left[1 - \frac{1}{(1.045)^{15}}\right]$

$\therefore$ P = 32,218.63

26. (a) $A = \frac{a}{i}\left[(1+i)^n - 1\right]$

$= \frac{150}{\frac{3.5}{100}}\left[\left(1+\frac{3.5}{100}\right)^{12} - 1\right]$

$= \frac{150}{0.035}\left[(1.035)^{12} - 1\right]$

$A = 2190.28$

27. (c) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$\therefore 10{,}000 = \frac{9}{0.04}\left[1 - \frac{1}{(1.04)^{30}}\right]$

$\therefore 10{,}000 = 9(17.2920)$

$\therefore a = 578.87$

28. (d) Value of $V = \frac{A}{i}\left[1 - \frac{1}{(1+i)^N}\right]$

Hence, A = 1200, n = 12 years, i = 0.08

$V = \frac{1200}{0.08}\left[1 - \frac{1}{(1.08)^{12}}\right]$

$V = 9043.29$

29. (a) $FV = \frac{a}{i}[(1+i)^n - 1]$

$= \frac{100}{0.05}[(1+0.05)^{10} - 1]$

$FV = 1258$

30. (b) $A = \frac{a}{i}[(1+i)^n - 1]$

$50{,}000 = \frac{a}{0.05}[(1+0.05)^{25} - 1]$

$\therefore 50{,}000 = \frac{a}{0.05}[2.3864]$

$\therefore a = 1047.62$

31. (b) $A = \frac{a}{i}[(1+i)^n - 1]$

$\therefore 3137.12 = \frac{100}{0.045}[(1+0.045)^n - 1]$

$\therefore 1.4117 = (1.045)^n - 1$

$\therefore n = 20$ years.

32. (a) $P = \frac{a}{i}\left[i - \frac{1}{(1+i)^n}\right]$

$\therefore 10{,}000 = \frac{1000}{0.05}\left[1 - \frac{1}{(1+0.05)^n}\right]$

$\therefore 0.5 = 1 - \frac{1}{(1.05)^n}$

$\therefore \frac{1}{(1.05)^n} = 0.5$

$\therefore (1.05)^n = 2$

$\therefore N = 14.2$ years

33. (b) $CI = P\left[\left(1 + \frac{R}{100}\right)^N - 1\right]$

$= 5120\left[\left(1 + \frac{12.5}{100}\right)^3 - 1\right]$

$\therefore CI = 2170$

34. (d) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$$20{,}000 = \frac{2000}{0.05}\left[1 - \frac{1}{(1+0.05)^n}\right]$$

$$\therefore 0.5 = 1 - \frac{1}{(1.05)^n}$$

$$\therefore \frac{1}{(1.05)^n} = 0.5$$

$$\therefore (1.05)^n = 2$$

$$\therefore n = 14.2 \text{ years}$$

35. (a) $A = \frac{a}{i}[(1+i)^n - 1](1+i)$

$$= \frac{500}{0.10}[1+0.10)^{12} - 1](1+0.10)$$

$$A = 11{,}761.36$$

36. (d) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$$= \frac{5000}{0.04}\left[1 - \frac{1}{(1+0.04)^{12}}\right]$$

$$P = 46{,}925.37$$

37. (c) $P = \frac{a}{i}$

$$= \frac{300}{0.10}$$

$$P = 3000$$

38. (b) $A = P\left(1+\frac{R}{100}\right)^N$

$\therefore 5200 = P\left(1+\frac{5}{100}\right)^6$

$\therefore P = 3880.$

39. (a) $CI = P\left[\left(1+\frac{R}{100}\right)^N - 1\right]$

$CI = 1000\left[\left(1+\frac{5}{100}\right)^4 - 1\right]$

$CI = 215.50$

40. (c) $A = P\left(1+\frac{R}{100}\right)^N$

$\therefore 2 = 1\left(1+\frac{5}{100}\right)^N$

$\therefore (1.05)^N = 2$

$\therefore N = 14.2$ year approved.

41. (d) $A = P\left(1+\frac{R}{100}\right)^N$

$\therefore 10{,}000 = P\left(1+\frac{4}{100}\right)^{18}$

$\therefore P = 4936.28$

42. Year $= \frac{114}{\text{Rate}}$

$= \frac{114}{8}$

Year = 14.25

$\approx$ 14.28 year.

43. (a) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{80}{0.05}\left[1 - \frac{1}{(1+0.05)^{20}}\right]$

P = 997.

44. (c) $P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$

$= \frac{4000}{0.05}\left[1 - \frac{1}{(1+0.05)^{25}}\right]$

P = 56375

Cash price = cash down + P

= 20,000 + 56,375

= 76,375

$\approx$ 76392

45. (a) $P = 3,00,000 - 2,00,000$

$P = 1,00,000$

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$1,00,000 = \frac{a}{\frac{12}{200}}\left[1 - \frac{1}{\left(1+\frac{12}{200}\right)^{20}}\right]$$

$$1,00,000 = \frac{a}{0.06}\left[1 - \frac{1}{(1.06)^{20}}\right]$$

$\therefore a = 8719.66$

46. (d) $S.I. = \frac{20,000 \times 5\% \times 4}{100} = 4000$

$C.I. = 20,000\ \{(1.05)^4 - 1\} = 4310.125$

$D = 310.125$

47. (d) $A = 10,000\ (1.03)^4\ (1.045)^2$

$= 12,290$

$\therefore$ C.I. = 2290

48. (d) $A = p(1+i)^N$

$10,000 = P\ (1.05)^2$

$= 9070$

49. (c) $A = P(1+i)^N$

$10,000 = P(1.025)^4$

$= 9059.50$

50. (d) Assuming simple interest

Let the amount = x

$$\frac{x}{\left(1+\frac{3.5\times 16}{100}\right)}+\frac{x}{\left(1+\frac{3.5\times 13}{100}\right)}+\frac{x}{\left(1+\frac{3.5\times 10}{100}\right)}=1L$$

$$\frac{x}{1.56}+\frac{x}{1.455}+\frac{x}{1.35}=1L$$

$x(0.6410 + 0.6873 + 0.7407) = 1\ L$

$x(2.0690) = 1\ L$

$x = \frac{100000}{2.0690}$ $\qquad$ $x = 48332$

51. (b) $2 = 1\ (1.05)^N$

For perfect answer taking log

$\log 2 = N \log 1.05$

$0.3010 = N(0.0212)$

$N = 14.2$

52. (c) $A = P\ (1+i)^N$

$3 = 1\ (1.025)^N$

Trial and Error

(c) 22 years 23 month

53. (c) $A = P(1+i)^N$

$9000 = 23240\ (0.9)^N$

$0.3873 = (0.9)^N$

N = 9 years [Trial and Error]

54. (d) $A = P(1 - i)^N$

$2,00,000 = 490,740\ (0.85)^N$

$0.4075 = (0.85)^N$

5 years 7 months [Trial and Error]

55. (d) $49074 = 490740\ (0.85)^N$

$0.1 = (0.85)^N$

N = 14 years 2 month

[Trial and Error]

SELF ASSESSMENT TEST 6
ANNUITIES

25 Question, 25 Marks

1. Find the amount of an annuity of Rs. 100 in 10 years allowing compound interest at 5%, given $(1.05)^{10} = 1.6289$.

   a) Rs. 1527.80 b) Rs. 1257.80

   c) Rs. 1357.90 d) None of the above

2. Find the present value of an annuity certain of Rs. 300 for 5 years reckoning compound interest at 4% per annum. Given $(1.04)^5 = 1.2167$

   a) Rs. 1335.78 b) Rs. 1353.87

   c) Rs. 1533.98 d) None of the above

3. Gargi borrows Rs. 20000 at 4% compound interest and agrees to pay both the principal and the interest in 10 equal annual instalments at the end of each year. Find the amount of these in-stalments. Given $(1.04)^{10} = 1.4802$.

   a) Rs. 2470 b) Rs. 2570 c) Rs. 2740 d) Rs. 2760

4. Find the amount of an annuity of Rs. 50 payable quarterly for 15 years at 5%, compounded quarterly. Given $(1.0125)^{60} = 2.1028$

   a) Rs. 4422 b) Rs. 4411 c) Rs. 5544 d) Rs. 4433

5. Find the present value of an annuity of Rs. 50 payable quarterly for 15 years at 5%, compound-ed quarterly. Given $(1.0125)^{60} = 2.1028$

   a) Rs. 2000 b) Rs. 2098 c) Rs. 2108 d) Rs. 2100

6. Find the value of perpetuity of Rs. 105 a year at 5.25% per annum.

   a) Rs. 2000 b) Rs. 2098 c) Rs. 2108 d) Rs. 2100

7. A wagon is purchased on instalment basis, such that Rs. 5000 is to be paid on the signing of the contract and four yearly instalment of Rs. 3000 each payable at the end of the first, second, third and fourth years. If interest is charged at 5% p.a., what would be the cash down price? Given $(1.05)^{-4} = 0.82270$

   a) Rs. 16358 b) Rs. 15638 c) Rs. 10638 d) Rs. 14500

8. A government constructed housing flat costs Rs. 136000; 40% is to be paid at the time of possession and the balance, reckoning compound interest of 9% p.a. is to be paid in 12 equal annual instalments. Find the amount of each such instalment. Given $(1.09)^{-12} = 0.3558$

a) Rs. 11000 b) Rs. 12000 c) Rs. 11400 d) Rs. 12400

9. What sum will buy an annuity of Rs. 1000 payable half-yearly for 5 years, the rate of interest being 8% p.a. compounded half-yearly? Given $(1.04)^{-10} = 0.6756$

a) Rs. 8000 b) Rs. 9000 c) Rs. 9110 d) Rs. 8110

10. Dipti borrowed Rs. 40000 at 6% compound interest promising to repay Rs. 9000 at the end of each of the first four years and to pay the balance at the end of the fifth year. Ascertain how much she would pay as the final instalment. Given $(1.06)^{-4} = 0.79206$

a) Rs. 8809 b) Rs. 11789 c) Rs. 12897 d) Rs. 10589

11. The present vale and amount of an annuity certain of Rs. 180 at a fixed rate per cent p.a. com-pound are Rs. 2000 and Rs. 3000 respectively. Find the rate of interest.

a) 4% b) 3.5% c) 5% d) 3%

12. A man retires at the age of 60 years and his employer gives him a pension of Rs. 3600 a year paid in half-yearly instalments for the rest of his life. If the expectation of his life is taken to be 10 years and interest is 6% per annum payable half-yearly, determine the present value of the pension. Given $(1.03)^{-20} = 0.55362$

a) Rs. 26,783 b) Rs. 28,768 c) Rs. 26,893 d) Rs. 28,763

13. A freehold estate was worth Rs. 50,000. If the annual rent of the property be Rs. 2,000, find the rate per cent p.a.

a) 3% b) 3.5% c) 4% d) 5%

14. A machine costs a company Rs. 52000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life-time, when its scrap realizes a sum of Rs. 2500 only. The price for the new model is estimated to be 25% higher than the price of the present one. Find what amount should be set aside every year out of profits for the sinking fund, if it accumulates at 3.5% p.a. compound. Given $(1.035)^{25} = 2.3659$

a) Rs. 1590 b) Rs. 1650 c) Rs. 1602 d) Rs. 1592

15. A man decides to deposit Rs. 10000 at the end of each year in a bank which pays 10% per annum compound interest. If the instalments are allowed to accumulate, what will be total accumulation at the end of 9 years? Given $(1.1)^9 = 2.2583$

a) Rs. 124000 b) Rs. 125000 c) Rs. 125830 d) Rs. 124930

16. The annual rent of a perpetual annuity is Rs. 4000. Find its value, the interest being compounded at 8% p.a.

a) Rs. 40000 b) Rs. 45000 c) Rs. 50000 d) None of the above

17. The value and annual rent of perpetuity are Rs. 12500 and Rs. 1000 respectively. Find the rate of compound interest.

a) 7% b) 8% c) 10% d) None of the above

18. Find the value of a deferred perpetuity of Rs. 500 p.a. to commence 10 years hence at 6% p.a. compound interest. Given $(1.06)^{10} = 1.791$

a) Rs. 4653 b) Rs. 4563 c) Rs. 4356 d) Rs. 4365

19. A loan of Rs. 10,000 is to be repaid in 30 equal annual instalments of Rs. X. Find X, if the CI charged is at the rate of 4% p.a. (Annuity is an annuity immediate). Given $(1.04)^{30} = 3.2434$

a) Rs. 878.80 b) Rs. 758.40 c) Rs. 578.40 d) Rs. 598.80

20. A man buys a house for Rs. 60,000 on condition that he will pay Rs. 30000 cash down and the balance in 10 equal annual instalments, the first to be paid one year after the date of purchase. Calculate the amount of each instalment, compound interest being computed at the rate of 5% p.a. Given $(1.05)^{-10} = 0.6139$

a) Rs. 5883 b) Rs. 3885 c) Rs. 8583 d) Rs. 3588

21. The annual subscription for the membership of a club is Rs. 25 and a person may become a life-member by paying Rs. 1000 in a lump sum. Find the rate per cent per annum.

a) 2% b) 3% c) 2.5% d) 3.5%

22. An overdraft of Rs. 50,000 is to be paid back in equal instalments over a period of 20 years. Find the value of the instalment, if interest is compounded annually at 14% per annum. Given $(1.14)^{20} = 13.74349$

a) Rs. 7549 b) Rs. 7561 c) Rs. 7571 d) Rs. 7539

23. What is the value of an annuity at the end of 5 years, if Rs. 100 per month is deposited into an account earning interest 9% per year compounded monthly? Given log10075 = 4.003245055 and antilog 4.1947033 = 15656.81067

a) Rs. 7542 b) Rs. 7892 c) Rs. 7598 d) Rs. 7498

24. A man borrows Rs. 20,000 at interest rate 4% per annum compounded annually and agrees to pay both the principal and the interest in 10 equal annual instalments at the end of each year. Find the amount of these instalments. Given log104 = 2.0170 and log6761 = 3.8300

a) Rs. 2400 b) Rs. 2470 c) Rs. 2489 d) Rs. 2459

25. Rs. 12,000 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the amount of this annuity after 10th payment? Given $(1.005)^{10} = 1.0511$

a) Rs. 122000 b) Rs. 120680 c) Rs. 122980 d) Rs. 122640

EXPLANATORY ANSWERS

1. A = 100/0.05[(1.05)^{10} − 1] = 2000(1.6289 − 1) = 1257.80, Option B

2. PV = 300/0.04[1-(1.04)$^{-5}$] = 7500[1 − 1/1.2167] = 1335.78. Option A

3. 20000 = P/0.04[1 − (1.04)$^{-10}$]
   800 = P[1 − 1/1.4802]
   P = (800 * 1.4802)/0.4802 = 2470
   Option A

4. Amount = 50/0.0125[(1.0125)^{60} − 1] = 4000(1.1028) = 4411. Option B

5. PV = 50/0.0125[1 − (1.0125)$^{-60}$] = 4000(1.1028/2.1028) = 2098. Option B

6. Perpetuity PV = 105/5.25% = 2000. Option A

7. PV of instalment = 3000/0.05[1 − 0.82270] = 10638
   Cash down price = 5000 + 10638 = 15638. Option B

8. Loan amount = 136000 * .60 = 81600
   81600 = P/0.09 [1 − 0.3558]
   7344 = P(0.6442)
   P = 11400. Option C

9. PV = 1000/0.04[1 − 0.6756] = 25000(0.3244) = 8110. Option D

10. PV of 4 instalments = 9000/0.06[1 − 0.79206] = 31191
    PV of loan amount remaining to be paid = 40000 − 31191 = 8809
    FV of Rs. 8809 payable after 5 years = 8809(1.06)^{5} = 11789
    Option B

11. $2000 = 180/r[1 - (1+r)^{-n}] = 180/r[(1+r)^n - 1]/(1+r)^n = 3000/(1+r)^n$
$3000 = 180/r[(1+r)^n - 1]$
$3/2 = (1+r)^n$
$3000 = 180/r[3/2 - 1]$
$r = 180/3000[1/2] = 0.03$
Interest rate = 3%
Option D

12. $PV = 1800/0.03[1 - (1.03)^{-20}] = 60000[1 - 0.55362] = 26783$. Option A

13. $R/100 = 2000/50000 = 0.04$. R = 4%. Option C

14. Amount required = 52000 * 1.25 − 2500 = 62500
$62500 = A/.035[(1.035)^{25} - 1] = A(39.0257)$
A = 62500/39.0257 = 1602. Option C

15. $FV = 10000/0.10 [(1.1)^9 - 1] = 100000[2.2583 - 1] = 125830$. Option C

16. Value = 4000/0.08 = 50000. Option C

17. R/100 = 1000/12500 = 0.08. R = 8%. Option B

18. Value = 500/0.06 = 8333
PV of Value = $8333/(1.06)^{10} = 8333/1.791 = 4653$. Option A

19. $10000 = X/0.04 [1 - 1/3.2434] = X(17.292)$
X = 10000/17.292 = 578.40. Option C

20. $30000 = X/0.05[1 - 0.6139] = X(7.722)$
X = 30000/7.722 = 3885. Option B

21. R/100 = 25/1000 = 0.025. R = 2.5%. Option C

22. $50000 = X/0.14[1 - 1/13.74349] = X(6.6231)$
X = 50000/6.6231 = 7549. Option A

23. Amount = $100/0.0075[(1.0075)^{60} - 1]$

$X = (1.0075)^{60}$

$\text{Log}X = 60 \log(1.0075) = 60[0.003245055] = 0.1947033 = \text{Log } 1.565681067$

$X = 1.565681067$

Amount = $13333.33(1.565681067 - 1) = 7542$. Option A

24. $20000 = P/0.04[1 - (1.04)^{-10}]$

Let X = (1.04)10

$\text{Log } X = 10 \log(1.04) = 10(0.0170) = 0.170 = 4 - 3.83 = \log 10000 - \log 6761 = \log 1.4791$

$X = 1.4791$

$20000 = P/0.04[1 - 1/1.4791] = P(8.098)$

$P = 20000/8.098 = 2470$

Option B

25. Amount = $12000/0.005[(1.005)^{10} - 1] = 2400000(0.0511) = 122640$. Option D

SELF ASSESSMENT TEST 7
INTEREST

24 Question, 24 Marks

1. Mahesh lent Rs. 100000, partly at 12% and partly at 10% simple interest. After three years he got Rs. 31500 as total simple interest. How much did he lend at the 12%?

   a) 25000 b) 75000 c) 50000 d) 40000

2. At what rate will a sum of money Rs. 205000, becomes Rs. 410000 with simple interest in 20 years?

   a) 5% b) 6.67% c) 5.5% d) 7.14%

3. A sum of money amounts to Rs. 44700 in 14 years at 8.33% simple interest. When will it double itself at the same rate?

   a) 16 years b) 12 years c) 11 years d) 17 years

4. A sum of Rs. 57000 is lent out in two parts A and B in such a way that interest on A @ 8% per annum simple interest for 5 years is equal to that on B at 0.5% per annum sim-ple interest for 15 years. Find the value of A.

   a) 9000 b) 6000 c) 48000 d) 45000

5. In what time will Rs. 4,500 amount to 22,500 at 4% per annum simple interest?

   a) 50 b) 60 c) 110 d) 100

6. Mr. M takes a loan of Rs. 525000 at 4% p.a. Compound Interest from Mr. J. He pays Mr. J Rs. 250000 at the end of 1st year. How much should he pay at the end of the 2nd year in order to clear his dues?

   a) 300000 b) 300800 c) 307840 d) 370480

7. A person bought a robot under the following scheme: Down payment of Rs. 150,000 and the rest amount at 8% per annum for 2 years. In this way, he paid Rs. 289,200 in total. Find the actual price of the robot. (Assume simple interest).

   a) 270000 b) 300000 c) 280000 d) 275000

8. Manohar Bhaiya borrows Rs. 140,000 at simple interest from the village co-operative society bank. At the end of 3 years, he again borrows Rs. 60,000 and closes his account after paying Rs. 92,300 as interest after 8 years from the time he made the first bor-rowing. What is the simple interest rate of interest charged by the bank?

a) 6.5% b) 5.5% c) 5% d) 7%

9. At R% per annum simple interest, simple interest on Rs. X for 20 years is equal to 4/9 th of its amount for that period. Find R.

a) 4% b) 5% c) 4.5% d) 6.5%

10. Rs. X lent out at compound interest amounts to Rs. 484000 in 2 years at 10% p.a. Find X.

a) 400000 b) 410000 c) 390000 d) 380000

11. A sum of money doubles itself in 5 years @ R% per annum compounded annually. In how many years will it become four times at the same compound rate on interest?

a) 10 b) 8 c) 12 d) 17

12. Rs. X doubles itself and becomes Rs. 2X at a certain rate of compound interest in 3 years. In how many years will the ratio of the principal to the compound interest be 1:3?

a) 6 b) 5 c) 8 d) 7.5

13. Rs. 1,25,000 placed at compound interest becomes Rs. 250000 in 3 years @ R% per annum compound interest? In how many years will it amount becomes 1000000?

a) 9 years b) 10 years c) 11 years d) 7 years

14. Rs. M grows up to Rs. 80000 in 2 yrs. and up to Rs. 85000 in 3 yrs @ R% per annum compound interest. Find the value of R.

a) 6.25% b) 6% c) 6.75% d) 5.5%

15. An amount of money grows upto Rs. 86400 in 2 years and upto Rs. 103680 in 3 years on compound interest. What is the sum?

a) Rs. 51000 b) Rs. 61000 c) Rs. 58000 d) Rs. 60000

16. The difference between Compound interest and simple interest on a certain sum of money is Rs. 400 for first two years and Rs. 1220 for first three years. Find the sum, if the rate is same in both the cases.

a) 6400 b) 8000 c) 12000 d) 9400

17. The Compound Interest @ R% per annum on a certain sum of money Rs. X for 2 yrs. is Rs. 357 and the Simple Interest on the same sum of money Rs. X at the same rate of in-terest is Rs. 350. What is the value of R?

a) 4% b) 5% c) 4.5% d) 5.5%

18. The value of a residential flat constructed at a cost of Rs. 15,00,000 is depreciating at the rate of 9% per annum. Find its value after two years of construction.

a) 12,42,150 b) 12,50,000 c) 12,75,850 d) 12,95,540

19. A money lender borrows a sum from market at 3% per annum simple interest and lent it out to another person at 6% per annum compounded half-yearly. If after one year, he gets a profit of Rs. 618, then find out the sum borrowed by the money lender.

a) Rs. 25,000 b) Rs. 20,000
c) Rs. 18,000 d) Can't be determined

20. The simple interest on a certain sum of money for 2 years at 12% per annum is Rs. 3120. What would be the compound interest at the same rate and for the same time?

a) Rs. 3507.80 b) Rs. 3407.20
c) Rs. 3207.20 d) Rs. 3307.20

21. What would be the compound interest on Rs. 10000 for three years, if the rate of inter-est is 5% for the first year, 6% for the second year and 7% for the third year?

a) Rs. 1900.10 b) Rs. 1919.10
c) Rs. 1909.10 d) Rs. 1809.10

22. Rs. 4000 becomes Rs. 5000 in 4 years at a certain rate of compound interest. What will be the sum at the end of 12 years?

a) Rs. 7812.50 b) Rs. 7612.50
c) Rs. 7712.50 d) Rs. 7512.50

23. Simple interest on a sum for 3 years at any rate of interest is Rs. 225 while compound interest on the same sum at the same rate for 2 years is Rs. 153. Find the sum and rate percent.

a) Rs. 1875, 4% b) Rs. 1875, 5%

c) Rs. 1785, 4% d) Rs. 1785, 5%

24. A man borrows Rs. 6000 at 10% compound rate of interest. At the end of each year he pays back Rs. 2000. How much amount should be pay at the end of third year to clear all his dues?

a) Rs. 3636 b) Rs. 3663 c) Rs. 3366 d) None of the above

EXPLANATORY ANSWERS

1. $X * 0.12 * 3 + (100000 - X) * 0.10 * 3 = 31500$
   $0.36X + 30000 - 0.30X = 31500$
   $0.06X = 1500$
   $X = 25000$; Option A

2. 410000/205000 = 2; Doubled; Rate = 100/20 = 5%. Option A

3. Time = 100/8.33 = 12 years. Option B

4. $A * 0.08 * 5 = (57000 - A) * 0.005 * 15$
   $0.4A = 4275 - 0.075A$
   $A = 4275 / 0.475 = 9000$; Option A

5. $22500 - 4500 = 18000 = 4500 * 0.04 * T$
   $T = 18000/180 = 100$ years; Option D

6. $525000 * 1.04 = 546000 - 250000 = 296000 \times 1.04 = 307840$. Option C

7. $150000 + X(1.16) = 289200$
   $X = 120000$; Actual price = 150000 + 120000 = 270000; Option A

8. $140000 * R/100 * 3 + 200000 * R/100 * 5 = 92300$
   $4200R + 10000R = 92300$
   $R = 6.5\%$; Option A

9. $X * R/100 * 20 = 4X (1 + 20R/100)/9$
   $9R/5 = 4 + 4R/5$
   $R = 4$, Option A

10. $484000 = X (1.1)^2$
    $X = 484000/1.21 = 400000$; Option A

11. Doubles = 5 years, Four times = 5 * 2 = 10 years. Option A

12. Double 3 years
    1:3 means Sum now becomes 4 times.
    Four times 3*2 = 6 years; Option A

13. Double 3 years
    Eight times = 3*3 = 9 years; Option A

14. 5000 = 80000 * R/100
    R = 5000/800 = 6.25%; Option A

15. (103680 – 86400) = 17280 = Interest on 86400 for 1 years
    R = 17280 / 86400 * 100 = 20%
    P = 86400 / (1.2)2 = 60000
    Option D

16. PII = 400
    PII (I/100 + 3) = 1220
    I/100 = 0.05
    I = 5%
    P (25/100) (25/100) =400
    P = 400 * 4 * 4 = 6400
    Option A

17. X * R/100 * R/100 = 7
    2X * R/100 = 350
    XR/100 = 175
    175 * R/100 = 7
    R = 4
    Option A

18. Value = 1500000 $(1 - 0.09)^2$ = 1242150.
    Option A

19. 3% pa CI is equivalent to $(1.03)^2 - 1 = 6.09\%$ pa SI
Profit = 3.09% = 618
100% = Sum borrowed = 618/3.09% = 20000. Option B

20. P = 3120 / 24% = 13000
CI = 13000 [$(1.12)^2 - 1$] = 13000 * 0.2544 = 3307.20.
Option D

21. CI = 10000 [(1.05)(1.06)(1.07) − 1] = 10000 * 0.19091 = 1909.10. Option C

22. In 4 years amount increases by 1000/4000 * 100 = 25%
In next 4 years 5000 shall become 5000 * 1.25 = 6250
In next 4 years 6250 shall become 6250 * 1.25 = 7812.50
Option A

23. Simple interest for 1 year = 225/3 = 75
Thus CI for 2 years = 75 + 75 + 75*R% = 153
75 * R/100 = 3
R = 300/75 = 4%
P = 225/12% = 1875; Option A

24. Year 1 − 6000 * 1.10 = 6600 − 2000 = 4600
Year 2 − 4600 * 1.1 = 5060 − 2000 = 3060
Year 3 − 3060 * 1.1 = 3366
Option C

4 SEQUENCES AND SERIES (PROGRESSIONS)

PART A - THEORY

- A sequence is defined as an array of numbers in such a manner so that there is a similarity in a given array, which enables us to determine the term or terms preceding or succeeding to such an array.

- A sequence can be categorized into 3 parts:
  a) Arithmetic Progression
  b) Geometric Progression
  c) Harmonic Progression

	Arithmetic Progression	Geometric Progression
Definition	Series which increases or decreases by a fixed quantity	Series which increases or decreases by a fixed proportion
First Term	a	a
Constant	Common Difference = d	Common Ratio = r
Last Term	$l = t_n = a + (n-1)d$	$l = t_n = a.r^{n-1}$
Sum	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}(a + l)$	$S_n = a.\frac{1-r^n}{1-r}$ when $r < 1$ $S_n = a.\frac{r^n - 1}{r - 1}$ when $r > 1$

- If three numbers are in G.P., their Logarithms are always in A.P.

Infinite GP Series

$$a + ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots\alpha = \frac{a}{1-r} \quad \text{given } |r| < 1$$

Sum of Natural Numbers:

$$\sum n = 1+2+3+.....+n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2+2^2+3^2+.....+n^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum n^3 = 1^3+2^3+3^3+.....+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

Harmonic Progression(H.P)

- Three numbers are in H.P, If their reciprocals are in A.P
- a,b,c are in H.P , if $\frac{1}{a}\ \frac{1}{b}\ \frac{1}{c}$ are in A.P.
- H.P fails when one of the terms of the A. P is Zero.

$$t_n \text{ of HP} = \frac{1}{t_n \text{ of the crresponding A.P}}$$

Concept of A.M , G.M and H.M

If a & b are any unequal real positive numbers then,

	A.M(A)	G.M(G)	H.M(H)
Definition	$\frac{a+b}{2}$	$+\sqrt{ab}$	$\frac{2ab}{a+b}$
Relation	i) A > ii) $A \times H$	G > $= G^2$	H

Things to remember

- The ratio of the sum of X number of A.Ms to the sum of Y number of A.Ms is always X : Y
- Two numbers can have more than one A.M/G.M/H.M
- A.Ms/G.Ms/ H.Ms are also the members of A.P/G.P/ H.P

CLASSWORK SECTION

ARITHMETIC PROGRESSION

Choose the most appropriate option (a), (b), (c) or (d).

1. The nth element of sequence 5, 7, 9, 11 . . . is

   (a) 3n + 2 (b) n + 4 (c) 2n + 3 (d) none of these

2. If −17, −13, −9, . . . in the progression then t_{10} =

   (a) 41 (b) 43 (c) 40 (d) 19

3. Which term of the progression −1, −3, −5, . . . is −57

   (a) 27^{th} (b) 29^{th} (c) 39^{th} (d) none of these

4. Which term of the series 7 + 11 + 15 + . . . is 467?

   (a) 116 (b) 190 (c) 119 (d) 125

5. The 10th term in $3, \frac{9}{2}, 6, \frac{15}{2}$ . . . is ____

   (a) 33 (b) $\frac{33}{2}$ (c) $-\frac{33}{2}$ (d) −33

6. If the ninth term of an AP is 30 then S_{17} = _____

   (a) 150 (b) 501 (c) 510 (d) 105

7. The a^{th} term of an AP is b and b^{th} term is a. Then c^{th} term of it is

   (a) a + b + c (b) b + a − 2c

   (c) a + b + c/2 (d) a + b − c

8. Third term of an AP is 8 and the 17th term is 51/2. The 23rd term is

   (a) 37 (b) 33 (c) 41 (d) 31

9. The n^{th} term of the series whose sum to n terms $3n^2 + 2n$ is

   (a) 3n − 1 (b) 8n − 2 (c) 11n − 3 (d) none of these

10. If 3 consecutive terms of AP are K + 2, 4K − 6 and K − 2 then k = _____

    (a) 1 (b) 2 (c) 3 (d) none of these

11. The 16th term of an AP is 99 and common difference is 8 then S_{21} is:
(a) 1230 (b) 1290 (c) 1239 (d) 1293

12. The sum of all odd numbers between 100 and 200 is
(a) 6200 (b) 6500 (c) 7500 (d) 3750

13. The sum of all positive integral multiples of 3 less than 100 is
(a) 1584 (b) 1665 (c) 1683 (d) none of these

14. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is
(a) 10200 (b) 15200 (c) 16200 (d) none of these

15. The sum of all numbers between 400 and 900 which are divisible by 13 is
(a) 22504 (b) 29405 (c) 25402 (d) 25350

16. The 4 arithmetic means between − 2 and 23 are
(a) 3, 13, 8, 18 (b) 18, 3, 8, 13
(c) 3, 8, 13, 18 (d) none of these

17. The r^{th} term of AP is (3r − 1)/6. The sum of first p terms of the series is
(a) n(3p + 1) (b) (p/12) (3p + 1)
(c) (p/12) (3p − 1) (d) none of these

18. A sum of Rs. 6240 is paid off in 30 installments such that each installment is Rs. 10 more than the preceding installment. The value of first installment is
(a) Rs. 36 (b) Rs. 30 (c) Rs. 60 (d) none of these

19. The 1st and the last term of an AP are −4 and 146. The sum of the terms is 7171. The number of terms
(a) 101 (b) 100 (c) 99 (d) none of these

PAST YEARS QUESTIONS

20. The sum of all natural numbers between 100 and 1000 which are multiple of 5 is
(a) 98450 (b) 96450 (c) 97450 (d) 95450

21. On 1st January every year a person buys national saving certificates of value exceeding that of his last years purchase by Rs. 100. After 10 years he finds that the total value of the certificates purchased by him is Rs. 54500. Find the value of certificates purchased by him in the first year

(a) 6000 (b) 4000 (c) 5000 (d) 5500

22. If in an AP, Tn represent nth term $t_7 : t_{10} = 5 : 7$ then $t_8 : t_{11}$ = _____

(a) 13 : 16 (b) 17 : 23 (c) 14 : 17 (d) 15 : 19

23. If sum of 3 arithmetic means between 'a' and 22 is 42 then 'a' = ____

(a) 14 (b) 11 (c) 10 (d) 6

24. If each month Rs. 100 increases in any sum then find out the total after 10 months if the sum of first month is Rs. 2000.

(a) 24500 (b) 24000 (c) 50000 (d) 60000

25. The 4th term of AP is three times the first term and the 7th term exceeds twice the third term by 1. Find the first term 'a' and common difference 'd'.

(a) a = 3, d = 2 (b) a = 4, d = 3

(c) a = 5, d = 4 (d) a = 6, d = 5

GEOMETRIC PROGRESSION

26. The last term of the series 0.5, 1, 2, 4 . . . to 8 term is

(a) 64 (b) 128 (c) 512 (d) none of these

27. Sum of three numbers x, y, z are in a GP is 39 and their product is 729. The values of x, y, z are

(a) 3, 27, 9 (b) 9, 3, 27 (c) 3, 9, 27 (d) none of these

28. If x, y, z are in GP, and xyz = 27/8. The value of y is

(a) 3/2 (b) 2/3 (c) 2/5 (d) none of these

29. The value of three numbers in GP, so that their sum is (57/2) and product is 729 are

(a) 2, 9, 27 (b) 6, 9, 27/2

(c) 4, 16, 64/3 (d) none of these

30. A ball is dropped from a height of 48 m and rebounds two third of the distance it falls. It continued to fall and rebound in this way, how far will it travel before coming to rest

(a) 240 m (b) 260 m (c) 380 m (d) none

31. If a, b, c are in GP, $a^p = b^q = c^r$ then 1/p, 1/q, 1/r are in

(a) AP (b) GP (c) HP (d) none of these

32. If x, y, z are pth, qth and rth terms of a GP then the value of $x^{q-r}\, y^{r-p}\, z^{p-q}$ is

(a) 0 (b) 1 (c) −1 (d) none of these

33. Let A be the A.M and G_1, G_2 be two GMs between two positive numbers. Then $G_1^3 + G_2^3$ is equal to

(a) $2AG_1G_2$ (b) AG_1G_2 (c) $(AG_1G_2)/2$ (d) none

34. If the pth term of the series 16, 8, 4, . . . is $\frac{1}{17}$. The value of p is

(a) 25 (b) 22 (c) 23 (d) none of these

35. If the first term and the common ratio of a GP are 1 and 1/2 respectively and sum of its n terms is equal to $\frac{255}{128}$. The value of n is

(a) 6 (b) 5 (c) 8 (d) none of these

36. If 'S' be the sum 'P' the product and 'R' the sum of the reciprocals of n terms in a GP then 'P' is . . .. of S^n and R^{-n}

(a) Arithmetic mean (b) Geometric mean
(c) Harmonic mean (d) none of these

37. The sum of 3 numbers in A.P. is 15. If 1, 4 and 19 added to them respectively the results are in G.P. The numbers are

(a) 26, 5, −16 (b) 2, 5, 8 (c) 5, 8, 2 (d) both (a) and (b)

38. Given x, y, z are in GP, $x^p = y^q = z^\sigma$ then $\frac{1}{p}, \frac{1}{q}, \frac{1}{\sigma}$ are in

(a) AP (b) GP
(c) Both AP and GP (d) none of these

39. If $x = a + a/r + a/r^2 + \ldots \infty$, $y = b - b/r + b/r^2 - \ldots \infty$; $z = c + c/r^2 + c/r^4 + \ldots \infty$, then value of $(xy)/z-(ab)/c$ is

(a) 0 (b) 1 (c) −1 (d) none of these

40. The value of $S = 2/3 + 5/9 + 2/27 + 5/81 + \ldots$ to infinite terms is

(a) 11/8 (b) 8/11 (c) 3/11 (d) none of these

41. The third term of GP is 2, the product of first five term is

(a) 2^5 (b) 2^3 (c) 5^3 (d) none of these

PAST YEARS QUESTIONS

42. Find n such that $\frac{a^{n+1}+b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b

(a) 1/2 (b) 1 (c) −1/2 (d) 0

43. If the first term of a GP exceeds the second term by 2 and the sum to infinity is 50; the series is

(a) $10, 8, \frac{32}{5} \ldots$ (b) $10, 8, \frac{5}{2}, \ldots$

(c) $10, \frac{10}{3}, \frac{10}{9}, \ldots$ (d) none

44. In a GP if the $(p + q)$th term is m and $(p - q)^{th}$ term is n then pth term is

(a) mn (b) $\sqrt{mn}$ (c) m^2 (d) n^2

45. If G be geometric mean between a and b then the value of $\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2}$ is equal to

(a) G^2 (b) $3G^2$ (c) $1/G^2$ (d) $2/G^2$

46. Find the product of 243, $243^{1/6}$, $243^{1/36}$, ... to ∞

(a) 1024 (b) 27 (c) 729 (d) 246

47. Geometric mean of p, p^2, p^3, ... p^n be

(a) p^{n+1} (b) $p^{\frac{n}{2}}$ (c) $p^{\frac{(n+1)}{2}}$ (d) none of these

48. A GP (Geometric Progression) consists of 2n terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in even places is S_2. The common ratio of the progression is

(a) n (b) $2S_1$ (c) $\frac{S_2}{S_1}$ (d) $\frac{S_1}{S_2}$

SPECIAL SERIES

49. The sum of 'n' term of the series 1x4 + 3x7 + 5x10 + _____

(a) $\frac{n}{2}[5n^2+4n-1]$ (b) $\frac{n}{2}[4n^2+5n-1]$

(c) $\frac{n}{2}[4n^2+5n+1]$ (d) None

50. 7+77+777+..... n terms is equal to

(a) $\frac{7}{9}[10^{n+1}-10]-\frac{7n}{9}$ (b) $\frac{7}{9}[10^{n+1}-10]+\frac{7n}{9}$

(c) $\frac{7}{81}[10^{n+1}-10]-\frac{7n}{9}$ (d) $\frac{7}{81}[10^{n+1}-10]+\frac{7n}{9}$

51. $\frac{1}{1x2}+\frac{1}{2x3}+\frac{1}{3x4}+$ ..... n terms is equal to

(a) $\frac{n}{2n+1}$ (b) $\frac{n}{n+1}$ (c) $\frac{1}{n+1}$ (d) None

52. Sum of 'n' terms whose t_n is $n^2 + 2^n$

(a) $\frac{n(n+1)(2n+1)}{6}+2(2^n-1)$ (b) $\frac{(n+1)(2n+1)}{6}+2(2^n-1)$

(c) $\frac{n(n+1)^2}{6}+2(2^n-1)$ (d) None

MIXED BAG

53. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, then 25th term of the A.P. is _____.

a) 1 b) 25 c) 0 d) −25

54. If the sum of p terms of an AP is same as the sum of its q terms, then the sum of the first (p + q) terms is:

a) 0 b) p + q c) p − q d) None of the above

55. If S_1, S_2, S_3 be the sums of n terms of three AP and the first term of each AP being 1 and the respective common difference are 1, 2, 3; then $S_1 + S_3$ = ?

a) S_2 b) $3S_2$ c) $0.5S_2$ d) $2S_2$

56. An AP consists of n terms. If the sum of its first three terms is x and the sum of the last three terms is y then the sum of all the terms of the AP is:

(a) $\frac{n}{6}(xy)$ (b) $\frac{n}{6}(x+y)$ (c) $n(x+y)$ (d) $\frac{n(x-y)}{6}$

57. 300 trees are planted in a regular pattern in rows in the shape of an isosceles triangle, the numbers in the successive rows diminishing by one from the base to the apex. How many trees are there in the row, which forms the base of the triangle

a) 30 b) 21 c) 27 d) 24

58. The first and the last term of an AP are "a" and "1" respectively. The sum of n^{th} term from the beginning and the n^{th} term from the end is:

a) a + 1 b) a − 1 c) a + 31 d) 2a + 1

59. If the sums of n, 2n and 3n terms of an AP be S_1, S_2 and S_3 respectively, then show that S_3 = ?

a) $3(S_2 - S_1)$ b) $(S_2 - S_1)$ c) $2(S_2 - S_1)$ d) $3(S_2 + S_1)$

60. If S_n be the sum of n consecutive terms of an AP, then the value of $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is:

a) 0 b) 1 c) 2 d) 3

61. The sum of first n terms of two AP are in the ratio (7n + 2) : (n + 4). Find the ratio of their 5th terms.

a) 1 : 5 b) 5 : 1 c) 2 : 3 d) 3 : 2

62. $31^3 + 32^3 + 33^3 + ... + 50^3$

a) 2010000 b) 3025000 c) 2870000 d) 1409400

63. The common ratio, last term, and the sum of a G.P. are 3, 486 and 728 respectively. The first term of the progression is:

a) 4 b) 6 c) 8 d) 2

64. When a certain golf ball is dropped on a piece of pavement, it bounces to a height of three-fifth the distance from which it falls. If the ball is dropped from a height of 100 cm, how far it has travelled when it hits the pavement for the tenth time ?

a) 397 cm b) 400 cm c) 450 cm d) 460 cm

65. If a, b, x, y, z are positive numbers such that a, x, b are in AP; a, y, b are in GP and (a + b)z = 2ab, then x, y, z are in:

a. Arithmetic Progression b. Geometric Progression
c. Harmonic Progression d. None of the above

66. If S_1, S_2, S_3 be respectively the sum of n, 2n and 3n terms of a GP, then $S_1 (S_3 - S_2) - (S_2 - S_1)^2$ is:

a) n b) 2n c) 3n d) 0

67. If "a" be the first term, "b" the nth term and "p" the product of the first n terms of a GP, then which of the following is true?

a) $p = ab$ b) $p = (ab)^n$
c) $p^2 = (ab)^n$ d) None of the above

68. The sum of 1st six terms of a G.P. is 9 times the sum of the first three terms. Find the common ratio.

a. 2 b. 3 c. 4 d. 8

69. The sum of the first three terms of a G.P. is to the sum of the first six terms as 125:152. Find the common ratio of the G.P.

a. 0.40 b. 0.50 c. 0.75 d. 0.60

70. The first, tenth and twenty-eighth term of an AP are three successive terms of a GP. Find the common ratio of the GP. given that the sum of the first 28 terms of the AP is 210, find its first term.

a. 2, 2 b. 2, 3 c. 3, 2 d. − 3, 2

71. An air pump used to extract air from a vessel removes one-tenth of the air at stroke each stroke. Find what fraction of original volume of air is left after the 5th stroke.

a) 0.54899 b) 0.54999 c) 0.59049 d) 0.60099

72. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, then $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in :

a. Harmonic Progression
b. Arithmetic Progression
c. Geometric Progression
d. None of the above

73. If a^2, b^2, c^2 are in AP, the $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in :

a. Geometric Progression
b. Arithmetic Progression
c. Both a) and b) above
d. None of the above

74. If a, b, c are in AP, then $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in:

a. Geometric Progression
c. None of the above
c. Arithmetic Progression
d. Harmonic Progression

75. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in AP, then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in :

a. Harmonic Progression
b. Geometric Progression
c. Arithmetic Progression
d. None of the above

76. Evaluate the following:

(i) $0.\overline{4}$ (ii) $0.\overline{42}$ (iii) $0.\overline{423}$ (iv) $0.4\overline{2}$
(v) $0.4\overline{23}$ (vi) $0.42\overline{3}$ (vii) $7.\overline{42}$

77. The ratio of the sum of x AM to y AM between two numbers is:

a) x : y b) $x^2 : y^2$ c) 1 : 1 d) None of the above

78. If a, b, c are in GP and x, y be the arithmetic means between a, b and b, c respectively, then which of the following/s is/are true?

(a) $\frac{a}{x}+\frac{c}{y}=2$ (b) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$
(c) Both a) and b) above (d) Neither a) nor b) is true

HOMEWORK SECTION

1. The nth element of the sequence 1, 3, 5, 7, ...is

   (a) n (b) 2n − 1 (c) 2n +1 (d) none of these

2. The nth element of the sequence −1, 2, −4, 8 ... is

   (a) $(-1)^n 2^{n-1}$ (b) 2^{n-1} (c) 2^n (d) none of these

3. $\sum_{i=4}^{7} \sqrt{2i-1}$ can ne written as

   (a) $\sqrt{7} + \sqrt{9} + \sqrt{11} + \sqrt{13}$ (b) $2\sqrt{7} + \sqrt{9} + 2\sqrt{11} + 2\sqrt{132}$

   (c) $2\sqrt{7} + \sqrt{9} + 2\sqrt{11} + 2\sqrt{132}$ (d) None of these

4. The sum to ∞ of the series -5,25,-125,625, ...can be written as

   (a) $\sum_{k=1}^{\infty}(-5)^k$ (b) $\sum_{k=1}^{\infty}(5)^k$ (c) $\sum_{k=1}^{\infty}-5^k$ (d) None of these

5. The first three terms of sequence when nth term t_n is $n^2 - 2n$ are

   (a) −1, 0, 3 (b) 1, 0, 2 (c) −1, 0, −3 (d) none of these

6. Which term of the progression −1, −3, −5, ... is −39

   (a) 21st (b) 20th (c) 19th (d) none of these

7. The value of x such that 8x + 4, 6x − 2, 2x + 7 will form an AP is

   (a) 15 (b) 2 (c) 15/2 (d) none of the these

8. The m^{th} term of an A. P. is n and n^{th} term is m. The rth term of it is

   (a) m + n +r (b) n + m − 2r (c) m + n + r/2 (d) m + n − r

9. The number of the terms of the series 10 + $9\frac{2}{3} + 9\frac{1}{3} + 9 +$..... will amount to 155 is

   (a) 30 (b) 31 (c) 32 (d) (a) and (b) both

10. The n^{th} term of the series whose sum to n terms is $5n^2 + 2n$ is

    (a) 3n − 10 (b) 10n − 2 (c) 10n − 3 (d) none of these

11. The 20th term of the progression 1, 4, 7, 10.....is

    (a) 58 (b) 52 (c) 50 (d) none of these

12. The last term of the series 5, 7, 9, ... to 21 terms is

(a) 44 (b) 43 (c) 45 (d) none of these

13. The last term of the A.P. 0.6, 1.2, 1.8, ... to 13 terms is

(a) 8.7 (b) 7.8 (c) 7.7 (d) none of these

14. The sum of the series 9, 5, 1, ... to 100 terms is

(a) −18,900 (b) 18,900 (c) 19,900 (d) none of these

15. The two arithmetic means between −6 and 14 is

(a) 2/3, 1/3 (b) $2/3, 7\frac{1}{3}$ (c) $-2/3, -7\frac{1}{3}$ (d) None of these

16. The sum of three integers in AP is 15 and their product is 80. The integers are

(a) 2, 8, 5 (b) 8, 2, 5 (c) 2, 5, 8 (d) 8, 5, 3

17. The sum of n terms of an AP is $3n^2 + 5n$. The series is

(a) 8, 14, 20, 26 (b) 8, 22, 42, 68

(c) 22, 68, 114, ... (d) none of these

18. The number of numbers between 74 and 25,556 divisible by 5 is

(a) 5,090 (b) 5,097 (c) 5,095 (d) none of these

19. The pth term of an AP is (3p − 1)/6. The sum of the first n terms of the AP is

(a) n (3n + 1) (b) n/12 (3n + 1)

(c) n/12 (3n − 1) (d) none of these

20. The arithmetic mean between 33 and 77 is

(a) 50 (b) 45 (c) 55 (d) none of these

21. The 4 arithmetic means between −2 and 23 are

(a) 3, 13, 8, 18 (b) 18, 3, 8, 13

(c) 3, 8, 13, 18 (d) none of these

22. The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3rd term of the AP is

(a) $6\frac{4}{11}$ (b) 6 (c) 4/11 (d) none of these

23. The sum of a certain number of terms of an AP series −8, −6, −4, ... is 52. The number of terms is

(a) 12 (b) 13 (c) 11 (d) none of these

24. The first and the last term of an AP are −4 and 146. The sum of the terms is 7171. The number of terms is

a) 101 (b) 100 (c) 99 (d) none of these

25. The sum of the series 3½ + 7 + 10½ + 14 + ... to 17 terms is

(a) 530 (b) 535 (c) 535 ½ (d) none of these

26. The 7^{th} term of the series 6, 12, 24, ...is

(a) 384 (b) 834 (c) 438 (d) none of these

27. t_8 of the series 6, 12, 24,... is

(a) 786 (b) 768 (c) 867 (d) none of these

28. t_{12} of the series −128, 64, −32, ...is

(a) − 1/16 (b) 16 (c) 1/16 (d) none of these

29. The 4th term of the series 0.04, 0.2, 1, ... is

(a) 0.5 (b) 1/2 (c) 5 (d) none of these

30. The last term of the series 1, 2, 4,.... to 10 terms is

(a) 512 (b) 256 (c) 1024 (d) none of these

31. The last term of the series 1, −3, 9, −27 up to 7 terms is

(a) 297 (b) 729 (c) 927 (d) none of these

32. The last term of the series x^2, x, 1, ... to 31 terms is

(a) x^{28} (b) 1/x (c) $1/x^{28}$ (d) none of these

33. The sum of the series −2, 6, −18, ... to 7 terms is

(a) −1094 (b) 1094 (c) − 1049 (d) none of these

34. The sum of the series 243, 81, 27, ... to 8 terms is

(a) 364 (b) $364\frac{13}{30}$ (c) $364\frac{1}{9}$ (d) None of these

35. The sum of the series $\frac{1}{\sqrt{3}} + 1\ \frac{3}{\sqrt{3}} + \ldots\ \ldots\ \ldots$ to 18 terms is

(a) $\frac{9841(1+\sqrt{3})}{\sqrt{3}}$ (b) 9841 (c) $\frac{9841}{\sqrt{3}}$ (d) None of these

36. The second term of a G P is 24 and the fifth term is 81. The series is

(a) 16, 36, 24, 54,.. (b) 24, 36, 53, ...

(c) 16, 24, 36, 54,.. (d) none of these

37. The sum of 3 numbers of a G P is 39 and their product is 729. The numbers are

(a) 3, 27, 9 (b) 9, 3, 27 (c) 3, 9, 27 (d) none of these

38. In a G. P, the product of the first three terms 27/8. The middle term is

(a) 3/2 (b) 2/3 (c) 2/5 (d) none of these

39. If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be

(a) ₹ 163 (b) ₹ 183 (c) ₹ 163.83 (d) none of these

40. Sum of n terms of the series 4 + 44 + 444 + ... is

(a) $4/9\ \{\ 10/9\ (10^n - 1\ ) - n\ \}$ (b) $10/9\ (10^n - 1\ ) - n$

(c) $4/9\ (10^n - 1\ )^{-n}$ (d) none of these

41. Sum of n terms of the series 0.1 + 0.11 + 0.111 + ... is

(a) $(1/9)\ \{n - (\ 1 - (\ 0.1\ )^n\ )\}$ (b) $(1/9)\ \{n - (1-(0.1)^n)/9\}$

(c) $n - 1 - (0.1)^n/9$ (d) none of these

42. The sum of the first 20 terms of a G. P is 244 times the sum of its first 10 terms. The common ratio is

(a) $\pm\sqrt{3}$ (b) ± 3 (c) $\sqrt{3}$ (d) None of these

43. Sum of the series 1 + 3 + 9 + 27 +...is 364. The number of terms is

(a) 5 (b) 6 (c) 11 (d) none of these

44. The product of 3 numbers in G P is 729 and the sum of squares is 819. The numbers are

(a) 9, 3, 27 (b) 27, 3, 9 (c) 3, 9, 27 (d) none of these

45. The sum of the series 1 + 2 + 4 + 8 + .. to n term

(a) $2^n - 1$ (b) 2n − 1 (c) $1/2^n - 1$ (d) none of these

46. The sum of the infinite GP 14, − 2, + 2/7, − 2/49, + ... is

(a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) None of these

47. The sum of the infinite GP 14, − 2, + 2/7, − 2/49, + ... is

(a) $4\frac{1}{12}$ (b) $12\frac{1}{4}$ (c) 12 (d) none of these

48. The sum of the infinite G. P. 1 - 1/3 + 1/9 - 1/27 +... is

(a) 0.33 (b) 0.57 (c) 0.75 (d) none of these

49. The number of terms to be taken so that 1 + 2 + 4 + 8 + will be 8191 is

(a) 10 (b) 13 (c) 12 (d) none of these

50. Four geometric means between 4 and 972 are

(a) 12, 36, 108, 324 (b) 12, 24, 108, 320

(c) 10, 36, 108, 320 (d) none of these

51. Three numbers are in AP and their sum is 21. If 1, 5, 15 are added to them respectively, they form a G. P. The numbers are

(a) 5, 7, 9 (b) 9, 5, 7 (c) 7, 5, 9 (d) none of these

52. The sum of $1 + 1/3 + 1/3^2 + 1/3^3 +$..... upto infinite

(a) 2/3 (b) 3/2 (c) 4/5 (d) none of these

53. The sum of the infinite series 1 + 2/3 + 4/9 + .. is

(a) 1/3 (b) 3 (c) 2/3 (d) none of these

54. The sum of the first two terms of a G.P. is 5/3 and the sum to infinity of the series is 3. The common ratio is

(a) 1/3 (b) 2/3 (c) − 2/3 (d) (b) & (c) both

55. If p, q and r are in A.P. and x, y, z are in G.P. then $x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$ is equal to

(a) 0 (b) −1 (c) 1 (d) none of these

56. The sum of three numbers in G.P. is 70. If the two extremes by multiplied each by 4 and the mean by 5, the products are in AP. The numbers are

(a) 12, 18, 40 (b) 10, 20, 40

(c) 40, 20, 10 (d) (b) & (c) both

57. The sum of 3 numbers in A.P. is 15. If 1, 4 and 19 be added to them respectively, the results are is G. P. The numbers are

(a) 26, 5, −16 (b) 2, 5, 8 (c) 5, 8, 2 (d) (a) & (b) both

58. Given x, y, z are in G.P. and $x^p = y^q = z^\sigma$, then $1/p$, $1/q$, $1/\sigma$ are in

(a) A.P. (b) G.P.

(c) Both A.P. and G.P. (d) none of these

59. If the terms 2x, (x+10) and (3x+2) be in A.P., the value of x is

(a) 7 (b) 10 (c) 6 (d) none of these

60. If A be the A.M. of two positive unequal quantities x and y and G be their G. M, then

(a) $A < G$ (b) $A > G$ (c) $A \geq G$ (d) $A \leq G$

61. The A.M. of two positive numbers is 40 and their G. M. is 24. The numbers are

(a) (72, 8) (b) (70, 10) (c) (60, 20) (d) none of these

62. Three numbers are in A.P. and their sum is 15. If 8, 6, 4 be added to them respectively, the numbers are in G.P. The numbers are

(a) 2, 6, 7 (b) 4, 6, 5 (c) 3, 5, 7 (d) none of these

63. The sum of four numbers in G. P. is 60 and the A.M. of the first and the last is 18. The numbers are

(a) 4, 8, 16, 32 (b) 4, 16, 8, 32 (c) 16, 8, 4, 20 (d) none of these

64. A sum of ₹ 6240 is paid off in 30 instalments such that each instalment is ₹ 10 more than the proceeding installment. The value of the 1st instalment is

(a) ₹ 36 (b) ₹ 30 (c) ₹ 60 (d) none of these

65. The sum of $1.03 + (1.03)^2 + (1.03)^3 + ...$ to n terms is

(a) $103\{(1.03)^n - 1\}$ (b) $103/3\{(1.03)^n - 1\}$

(c) $(1.03)^n - 1$ (d) none of these

66. If x, y, z are in A.P. and x, y, (z + 1) are in G.P. then

(a) $(x - z)^2 = 4x$ (b) $z^2 = (x - y)$

(c) $z = x - y$ (d) none of these

67. The numbers x, 8, y are in G.P. and the numbers x, y, −8 are in A.P. The value of x and y are

(a) (−8, −8) (b) (16, 4) (c) (8, 8) (d) (a) & (b) both

68. The nth term of the series 16, 8, 4,..... in $1/2^{17}$. The value of n is

(a) 20 (b) 21 (c) 22 (d) none of these

69. The sum of n terms of a G.P. whose first terms is 1 and the common ratio is 1/2 , is equal to $1\frac{127}{128}$. The value of n is

(a) 7 (b) 8 (c) 6 (d) none of these

70. $t_4 = x$, $t_{10} = y$ and $t_{16} = z$. are in G.P., Then

(a) $x^2 = yz$ (b) $z^2 = xy$ (c) $y^2 = zx$ (d) none of these

71. If x, y, z are in G.P., then

(a) $y^2 = xz$ (b) $y(z^2 + x^2) = x(z^2 + y^2)$

(c) $2y = x+z$ (d) none of these

72. The sum of all odd numbers between 200 and 300 is

(a) 11,600 (b) 12,490 (c) 12,500 (d) 24,750

73. The sum of all natural numbers between 500 and 1000 which are divisible by 13, is

(a) 28,405 (b) 24,805 (c) 28,540 (d) none of these

74. If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, ... the resulting sum is

(a) 'a' perfect cube (b) 'a' perfect square

(c) 'a' number (d) none of these

75. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is

(a) 10,200 (b) 15,200 (c) 16,200 (d) none of these

76. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 and 5 is

(a) 2,200 (b) 2,000 (c) 2,220 (d) none of these

77. A person pays ₹ 975 by monthly instalment each less then the former by ₹ 5. The first instalment is ₹ 100. The time by which the entire amount will be paid is

(a) 10 months (b) 15 months (c) 14 months (d) none of these

78. A person saved ₹ 16,500 in ten years. In each year after the first year he saved ₹ 100 more than he did in the preceding year. The amount of money he saved in the 1st year was

(a) ₹ 1000 (b) ₹ 1500 (c) ₹ 1200 (d) none of these

79. At 10% C.I. p.a., a sum of money accumulate to ₹ 9625 in 5 years. The sum invested initially is

(a) ₹ 5976.37 (b) ₹ 5970 (c) ₹ 5975 (d) ₹ 5370.96

80. The population of a country was 55 crores in 2005 and is growing at 2% p.a C.I. the population is the year 2015 is estimated as

(a) 57.05 (b) 60.05 (c) 67.00 (d) none of these

HOMEWORK SOLUTIONS

1. (c) Here a = 3, cd = d = 3 − 1 = 2

T(n) = a + (n − 1)d = 3 + (n − 1)2 = 2n + 1;

2. (a) Here a = - 1, cr = r = 2/-1 = -2

$T(n) = a.r^{n-1} = (-1).(-2)^{n-1} = (-1).(-1)^{n-1}.2^{n-1} = (-1)^n.2^{n-1}$

3. (a)

$$\sum_{i=4}^{7}\sqrt{2i-1} = \sqrt{7}+\sqrt{9}+\sqrt{11}+\sqrt{13}$$

4. (a) Here a = - 5, common ratio = r = 25/ - 5 = -5

So the terms are: $(-5), (-5)^2, (-5)^3, (-5)^4, \ldots\ldots, (-5)^k, \ldots$

Sum to infinity can be written as: $\sum_{k=1}^{\alpha}(-5)^k$

5. (a) $T(n) = n^2 - 2n$

$T(1) = 1^2 - 2(1) = -1$

$T(2) = 2^2 - 2(2) = 0$

$T(3) = 3^2 - 2(3) = 3$

6. (b) Here, a = -1, cd = -3 − (-1) = - 2, T(n) = - 39 = L

$$N = \frac{L-a}{d}+1 = \frac{-39-(-1)}{-2}+1 = \frac{-38}{-2}+1 = 19+1 = 20$$

Value of 20th term is − 39.

7. (c) Three numbers a, b, c are in AP, i.e., 2b = a + c

2 (6x − 2) = (8x + 4) + (2x + 7) = 9x + 11

Or, 2x = 11 + 4 = 15

Thus, x = 15/2.

8. (d) For a AP series, if T(A) = B and T(B) = A, then T(C) = A + B − C

So, if T(m) = n & T(n) = m, the rth term = T(r) = m + n − r

9. (d) A = 10, D = 29/3 − 10 = - 1/3, S(n) = 155

$$155=\frac{n}{2}[2A+(n-1)D]=\frac{n}{2}\left[20-\frac{n-1}{3}\right]=\frac{n}{6}[61-n]$$

$$Or, 930=n(61-n);\therefore n^2-61n+930=0,(n-30)(n-31)=0$$

n = 30 or 31.

10. (c) $S(n) = 5n^2 + 2n$

S(1) = T(1) = 5 + 2 = 7 = A

S(2) = 20 + 4 = 24 = T(1) + T(2). T(2) = 24 − 7 = 17

CD = D = T(2) − T(1) = 17 − 7 = 10

T(n) = A + (n − 1)D = 7 + (n − 1)10 = 10n − 3

11. (a) A = 1, CD = 4 − 1 = 3, T(20) = A + 19D = 1 + 19(3) = 58.

12. (c) A = 5, D = 7 − 5 = 2, T(21) = A + 20D = 5 + 40 = 45.

13. (b) A = 0.6, D = 1.2 − 0.6 = 0.6, T(13) = A + 12D = 0.6 + 7.2 = 7.8.

14. (a) A = 9, D = 5 − 9 = - 4, S(100) = 100/2[18 − 99(4)] = 50[- 378] = - 18900.

15. (b) A = - 6, T(4) = 14 = A + 3D. Thus D = 20/3

The two AM are (-6 + 20/3) = 2/3 and (2/3 + 20/3) = 22/3 = 7 1/3

16. (c) or (d)

Option selection shall also help. Both (c) and (d) sum up to 15 and product is 80 and also are in AP. Both options are correct.

Or, one can take numbers (A − D), A, (A + D), and solve the following to equations to find the value of A and D.

(1) [A + D + A + A − D] = 15

(2) (A − D).A.(A + D) = 80

On solving one shall get numbers, 2-5-8 or 8-5-2.

17. (a) $S(n) = 3n^2 + 5n$

$S(1) = T(1) = 3 + 5 = 8$

$S(2) = 12 + 10 = 22 = T(1) + T(2)$, $T(2) = 22 - 8 = 14$

$CD = D = T(2) - T(1) = 14 - 8 = 6$

The AP series is: 8, 14, 20, 26, ...

18. (b) A= 75, L = 25555, D = 5

$$N = \frac{L-A}{D}+1 = \frac{25555-75}{5}+1 = \frac{25480}{5}+1 = 5097$$

19. (b) $T(p) = 1/6\ [3p - 1]$

$$S(n) = \sum_{p=1}^{n} T_p = \frac{1}{6}\left[3\sum n - n\right] = \frac{1}{6}\left[\frac{3n(n+1)}{2} - n\right] = \frac{1}{12}\left[3n^2 + n\right] = \frac{n}{12}(3n+1)$$

(c) AM between 33 and 77 = (33 + 77)/2 = 110/2 = 55.

21. (c) A = - 2, T(6) = 23 = A + 5D, D = 25/5 = 5

The 4 AM between -2 and 23 are:

(- 2 + 5)= 3; (3 + 5) = 8; (8 + 5) = 13; (13 + 5) = 18;

(a) A = 14

S(5) = - S(10)

$$\frac{5}{2}[28+4D] = -\frac{10}{2}[28+9D]$$

$$140 + 20D = -280 - 90D$$

$$110D = -420; D = -42/11$$

T(3) = A + 2D = 14 − 84/11 = 70/11 = 6 4/11.

23. (b) A = - 8, D = - 6 + 8 = 2, L = S(N) = 52

$$52 = \frac{N}{2}[-16 + (N-1)2]$$

$$104 = N(2N - 18)$$

$$52 = N(N - 9)$$

Or, $N^2 - 9N - 52 = 0$

Or, (N − 13)(N + 4) = 0

N = 13;

24. (a) 7171 = N/2(-4 + 146) = 71N

N = 7171 / 71 = 101.

25. (c) A = 3.5, D = 7 – 3.5 = 3.5

S(17) = 17/2[7 + 16(3.5)] = 535.5.

26. (a) A = 6, R = 12/6 = 2. $T_7 = A.R^6 = 6(2)^6 = 384$.

27. (b) A = 6, R = 12/6 = 2. $T_8 = A.R^7 = 6(2)^7 = 768$.

28. (c) A = - 128, R = 64/-128 = - ½

$T_{12} = A.R^{11} = (-128).(-½)^{11} = 2^7/2^{11} = ½^4 = 1/16$.

29. (c) A = 0.04, R = 0.2/0.04 = 5. $T_4 = A.R^3 = 0.04(5)^3 = 5$.

30. (a) A = 1, R = 2/1 = 2, $T_{10} = A.R^9 = 1.(2)^9 = 512$.

31. (b) A = 1, R = -3/1 = -3. $T_7 = A.R^6 = (1).(-3)^6 = 729$.

32. (c) $A = x^2$, $R = x/x^2 = 1/x$, $T_{31} = A.R^{30} = x^2(1/x)^{30} = 1/x^{28}$.

33. (a) A = - 2, R = 6/-2 = -3

$$S_7 = (-2)\left[\frac{1-(-3)^7}{1-(-3)}\right] = \frac{-2}{4}\left(1+3^7\right) = -1094$$

34. (d) A = 243, R = 81/243 = 1/3

$$S_8 = (243)\left[\frac{1-\frac{1}{3^8}}{1-\frac{1}{3}}\right] = 3^5\left[\frac{3^8-1}{3^8}.\frac{3}{2}\right] = \frac{3^8-1}{18} = \frac{6560}{18} = \frac{3280}{9} = 364\frac{4}{9}$$

35. (d) A = 1/√3, R = 1/A = √3

$$S_{18} = \left(\frac{1}{\sqrt{3}}\right)\left[\frac{\left(\sqrt{3}\right)^{18}-1}{\sqrt{3}-1}\right] = \left(\frac{1}{\sqrt{3}}\right)\left[\frac{19682}{\sqrt{3}-1}\right]\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)} = \frac{(9841)\left(\sqrt{3}+1\right)}{\sqrt{3}}$$

(c) $T_2 = A.R = 24$; $T_5 = A.R^4 = 81$

$R^3 = T_5/T2 = 81/24 = (27/8) = (3/2)^3$

$R = 3/2$

$A = 24/R = 24/1.5 = 16$

Series: 16, 24, 36, 54, 81, ...

37. (d) Options can be used.

a) 3, 27, 9 are not in GP. Rejected

b) 9, 3, 27 are not in GP, rejected

c) 3, 9, 27 are in GP. Sum = 39, Product = 729

But again, for 27, 9, 3, which are also in GP, same Sum = 39 and Product = 729 exists.

We thus have 2 set of answer: (3 – 9 – 27) & (27 – 9 – 3).

38. (a) Product of three numbers in GP = 27/8. Let the middle term is A.

$A^3 = 27/8 = (3/2)^3$

$A = 3/2$.

39. (c) $A = 1$, $R = 2/1 = 2$, $N = 14$

$$S_{14} = (1)\left[\frac{2^{14}-1}{2-1}\right] = 16383 = Rs.163.83$$

40. (a) $S(3) = 4 + 44 + 444 = 492$

Putting n= 3 in the options, the option which gives result 492 is the correct option.

(a) $4/9\ [10/9(1000 - 1) - 3] = 492$

41. (b) $S(3) = 0.1 + 0.11 + 0.111 = 0.321$

Putting n= 3 in the options, the option which gives result 0.321 is the correct option.

(a) $1/9\{3 - (1 - (0.1)^3)\} = 0.222$

(b) $1/9\{3 - (1 - (0.1)^3)/9\} = 0.321$

42. (a) S(20) = 243 S(10)

$$a\left[\frac{r^{20}-1}{r-1}\right]=244.a\left[\frac{r^{10}-1}{r-1}\right]$$

$$\left(r^{10}+1\right)=244$$

$r^{10} = 243 = (\sqrt{3})^{10}$; $r = \pm\sqrt{3}$

43. (b) A = 1, R = 3/1 = 3, S_n = 364

$$364=(1).\left[\frac{3^{n}-1}{3-1}\right]$$

$3^n = 729 = 3^6$; n = 6.

44. (d) Using options we get, 3,9,27 and 27,9,3 both fits in the given criteria.

45. (a) A = 1, R = 2/1 = 2, S(n) = $1(2^n - 1)/(2 - 1) = (2^n - 1)$.

46. (b) A = 14, R = -2/14 = -1/7, Sum to infinity = 14/(1 + 1/7) = 98/8 = 49/4 = 12 ¼.

47. (b) A = 14, R = -2/14 = -1/7, Sum to infinity = 14/(1 + 1/7) = 98/8 = 49/4 = 12 ¼.

48. (c) A = 1, R = -1/3 / 1 = -1/3, Sum to infinity = 1/(1 + 1/3) = ¾ = 0.75.

49. (b) A = 1, R = 2/1 = 2, S(n) = 8191

$$8191=(1).\left[\frac{2^{n}-1}{2-1}\right]$$

$2^n = 8192 = 2^{13}$. Thus, n = 13.

50. (a) A = 4, $T_6 = A.R^5 = 4.R^5 = 972$. Thus R = 3.
Four geometric means are: (4*3 = 12), (12*3 = 36), (36*3 = 108), (108*3=324)

51. (a) Of the options given, 5-7-9 are in AP. Again 5 + 7 + 9 = 21.
(5+1), (7+5), (9+15) = 6, 12, 24 and these are in GP.
Thus required numbers are 5,7,9.

52. (b) $A = 1, R = 1/3, S(\infty) = 1/(1 - 1/3) = 3/2.$

53. (b) $A = 1, R = 2/3, S(\infty) = 1/(1 - 2/3) = 3.$

54. (d) $A + AR = A(1 + R) = 5/3$

$A/(1 - R) = 3$

$(1 + R)(1 - R) = (1 - R^2) = 5/9$

$R^2 = 4/9 = (2/3)^2$

$R = \pm 2/3$

55. (c) AP: p(2), q(3), r(4)

GP: x(2), y(4), z(8)

$x^{q-r}.y^{r-p}.z^{p-q} = 2^{-1}.4^2.8^{-1} = 16/16 = 1$

56. (d) 10, 20, 40 are in GP. 40, 100, 160 are in AP. Option B fits in

Again, 40, 20, 10 are also in GP. 160, 100, 40 are also in AP. Option C also fits in.

57. (b) 2, 5, 8 are in AP and sum is 15. (2 + 1), (5 + 4), (8 + 19) = 3, 9, 27 are in GP

Again, 8, 5, 2 are also in AP. (8 + 1), (5 + 4), (2 + 19) = 9, 9, 21 are not in GP

58. (a) x, y, z are in GP. $y^2 = xz$

$x^p = y^q = z^r = k$

$k^{2/q} = k^{1/p}.k^{1/r}$

$2/q = 1/p + 1/r$

Thus, 1/p, 1/q, 1/r are in AP

59. (c) $2(x + 10) = 2x + 3x + 2$

$2x + 20 = 5x + 2$

$x = 18/3 = 6;$

60. (b) For unequal quantities, AM > GM.

61. (a) $A = 40, G = 24, A^2 - G^2 = 40^2 - 24^2 = 1024$

$X = A + \sqrt{(A^2 - G^2)} = 40 + 32 = 72$

$Y = A - \sqrt{(A^2 - G^2)} = 40 - 32 = 8$

62. (d) 3, 5, 7 are in AP, and sum = 15.

(3 + 8), (5 + 6), (7 + 4) = 11, 11, 11. But this is not in GP.

63. (d) 4, 8, 16, 32 are in GP. Sum = 4 + 8 + 16 + 32 = 60

AM of 4 and 32 = 36/2 = 18

But again, 32, 16, 8, 4 are also in GP and fits in the criteria.

64. (d) A = A, D = 10, N = 30, S(30) = 6240

$6240 = 30/2 [2A + 290]$

$416 = 2A + 290$

$A = 63;$

65. (b) Series is: $(1.03)^1, (1.03)^2, (1.03)^3,...$

$A = 1.03, R = 1.03$

$S_n = (1.03) [(1.03)^n - 1]/(1.03 - 1) = 103/3 [1.03^n - 1]$

66. (a) $2y = (x + z)$ and $y^2 = x(z + 1)$

$(x+z)^2/4 = x(z + 1)$

$x^2 + z^2 + 2xz = 4xz + 4x$

$x^2 + z^2 - 2xz = 4x$

$(x - z)^2 = 4x$

67. (b) $xy = 64; 2y = (x - 8)$

$(2y + 8)y = 64$

$2y^2 + 8y - 64 = 0$

$y^2 + 4y - 32 = 0$

$(y + 8) (y - 4) = 0$

$y = 4, -8$

$x = 64/4 = 16, 64/-8 = -8$

$(x, y) = (16, 4), (-8, -8)$

But, -8, -8, -8 is not in AP.

68. (c) $A = 16, R = 8/16 = ½, T_n = ½17 = 16(1/2)^{n-1} = 2^{5-n}$

$5 - n = - 17$

$22 = n$

69. (b) A = 1, CR = ½, S_n = 1 127/128

$$\frac{255}{128} = (1).\left[\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right] = \frac{2^n-1}{2^n}.\frac{2}{1} = \frac{2^n-1}{2^{n-1}}$$

$$\frac{256-1}{128} = \frac{2^8-1}{2^7} = \frac{2^n-1}{2^{n-1}}$$

n = 8

70. (c) T_4, T_{10}, T_{16} of a GP are also in GP

$Y^2 = X.Z$

71. (a) If X, Y, Z are in GP, $Y^2 = XZ$.

72. (c) A = 201, L = 299, D = 2, N = (299-201)/2 + 1 = 50

Sum = 50/2(201 + 299) = 12500

73. (a) A = 507, D = 13, L = 988, N = (988 – 507)/13 + 1 = 38

Sum = 38/2 [507 + 988] = 28405

74. (b) We know, 1 + 3 + 5 + 7 + ... nth term = n^2

Thus, when 1 is added to the sum of (3, 5, 7, ...), the resulting term is a perfect square.

75. (c) Divisible by 4

A = 100, D = 4, L = 300, N = (300 – 100)/4 + 1 = 51

Sum = 51/2 (100 + 300) = 10200

Divisible by 5

A = 100, D = 5, L = 300, N = (300 – 100)/5 + 1 = 41

Sum = 41/2 [100 + 300] = 8200

Divisible by both 4 and 5, i.e. 20

A = 100, D = 20, L = 300, N = (300 – 100)/20 + 1 = 11

Sum = 11/2(100 + 300) = 2200

Required sum = 10200 + 8200 – 2200 = 16200.

76. (a) Divisible by both 4 and 5, i.e. 20

A = 100, D = 20, L = 300, N = (300 − 100)/20 + 1 = 11

Sum = 11/2(100 + 300) = 2200

77. (b) A = 100, D = - 5. Sn = 975

975 = n/2 [200 − (n − 1)5]

1950 = n(205 − 5n)

390 = n(41 − n)

N2 − 41N + 390 = 0

(N − 26)(N − 15) = 0

N = 15

78. (c) A = A, D = 100, N = 10, Sum = 16500

16500 = 10/2 [2A + 900]

A = 1200.

79. (a) Sum invested = 9625 $(10/11)^5$ = 5976.37.

80. (d) P(2015) = 55 $(1.02)^{10}$ = 67.05 Crores

MIXED BAG (HOMEWORK)

1. If the p^{th} term of an A.P. is q and the q^{th} term is p the value of the $(p + q)^{th}$ term is_____.

   (a) 0 (b) 1 (c) −1 (d) None

2. If S_1 , S_2 , S_3 be the respectively the sum of terms of n, 2n, 3n an A.P. the value of $S_3 \div (S_2 - S_1)$ is given by _____.

   (a) 1 (b) 2 (c) 3 (d) None

3. The sum of n terms of two A.P.s are in the ratio of (7n−5)/(5n+17) . Then the _____ term of the two series are equal.

   (a) 12 (b) 6 (c) 3 (d) None

4. If a, b, c are in A.P. then the value of $(a^3 + 4b^3 + c^3)/[b(a^2 + c^2)]$ is

   (a) 1 (b) 2 (c) 3 (d) None

5. If a, b, c are in A.P. then the value of $(a^2 + 4ac + c^2)/(ab + bc + ca)$ is

   (a) 1 (b) 2 (c) 3 (d) None

6. The Pth term of an A.P. is 1/q and the qth term is 1/p. The sum of the pq terms is _____

   (a) $\frac{1}{2}(pq + 1)$ (b) $\frac{1}{2}(pq - 1)$ (c) pq+1 (d) pq−1

7. The sum of p terms of an A.P. is q and the sum of q terms is p. The sum of p + q terms is _____.

   (a) − (p + q) (b) p + q (c) $(p - q)^2$ (d) $p^2 - q^2$

8. If S_1, S_2, S_3 be the sums of n terms of three A.P.s the first term of each being unity and the respective common differences 1, 2, 3 then $(S_1 + S_3) / S_2$ is _____.

   (a) 1 (b) 2 (c) −1 (d) None

9. $2^{4n} - 1$ is divisible by

   (a) 15 (b) 4 (c) 6 (d) 64

10. $3^n - 2n - 1$ is divisible by

(a) 15 (b) 4 (c) 6 (d) 64

11. The least value of n for which the sum of n terms of the series $1 + 3 + 3^2 + \ldots\ldots$ is greater than 7000 is _____.

(a) 9 (b) 10 (c) 8 (d) 7

12. If 'S' be the sum, 'P' the product and 'R' the sum of the reciprocals of n terms in a G.P. then 'P' is the _____ of S^n and R^{-n}.

(a) Arithmetic Mean (b) Geometric Mean

(c) Harmonic Mean (d) None

13. If $1+a+a^2+\ldots\ldots\infty = x$ and $1+b+b^2+\ldots\ldots\infty = y$ then $1 + ab + a^2b^2 + \ldots\ldots\infty$ is given by _____.

(a) $(xy)/(x+y-1)$ (b) $(xy)/(x-y-1)$

(c) $(xy)/(x+y+1)$ (d) None

14. If a, b, c are in G.P. then the value of $a(b^2+c^2)-c(a^2+b^2)$ is _____

(a) 0 (b) 1 (c) –1 (d) None

15. If a, b, x, y, z are positive numbers such that a, x, b are in A.P. and a, y, b are in G.P. and $z=(2ab)/(a+b)$ then

(a) x, y, z are in G.P. (b) $x \geq y \geq z$

(c) both (d) None

16. If a, b-a, c-a are in G.P. and $a=b/3=c/5$ then a, b, c are in

(a) A.P. (b) G.P. (c) H.P. (d) None

17. If $S_1, S_2, S_3, \ldots\ldots S_n$ are the sums of infinite G.P.s whose first terms are 1, 2, 3 .....n and whose common ratios are 1/2, 1/3, ...1/(n+1) then the value of $S_1, S_2, S_3, \ldots\ldots S_n$ is

(a) (n/2) (n+3) (b) (n/2) (n+2)

(c) (n/2) (n+1) (d) n2 /2

MIXED BAG (HOMEWORK SOLUTION)

1. (a) T(1) = 2, T(2) = 1; A = 2, D = -1. T(3) = A + 2D = 2 − 2 = 0
   Similarly, $(p + q)^{th}$ term in this case = 0.

2. (c) Let n = 1. The three terms of AP = 100, 200, 300
   S1 = 100, S2 = 100 + 200 = 300; S3 = 100+200+300 = 600
   S3 / (S2 − S1) = 600/200 = 3;

3. (b) Sum of n terms of two AP are in the ratio (7n − 5)/(5n + 17)
   Equate ratio to 1, we get 7n − 5 = 5n + 17; n = 11
   Required term is (11 − 1)/2 + 1 = 6^{th} term

$$\frac{S_{n1}}{S_{n2}} = \frac{\frac{n}{2}[2A_1 + (n-1)D_1]}{\frac{n}{2}[2A_2 + (n-1)D_2]} = \frac{A_1 + \left(\frac{n-1}{2}\right)D_1}{A_2 + \left(\frac{n-1}{2}\right)D_2}$$

4. (b) A, B, C are in AP. Let the numbers be 1, 2, 3
   $(A^3 + 4B^3 + C^3) = (1 + 12 + 27) = 40$
   $B(A^2 + C^2) = 2(1 + 9) = 20$
   Required values = 40/20 = 2;

5. (b) A, B, C are in AP. Let the numbers be 1, 2, 3
   $(A^2 + 4AC + C^2) = (1 + 12 + 9) = 22$
   (AB + BC + CA) = (2 + 6 + 3) = 11
   Required value = 22/11 = 2;

6. (a) T2 = 1/3, T3 = 1/2.
   D = ½ - 1/3 = 1/6, A = 1/3 − 1/6 = 1/6
   (P.Q) = 6
   S_6 = 6/2 (2/6 + 5/6) = 7/2
   Option (a): (PQ + 1)/2 = 7/2

7. (a) S1 = 3, S3 = 1 (P = 1, Q = 3)

A = 3

A + A + D + A + 2D = 1; 3A + 3D = 1; D = -8/3

S4 = 4/2 [6 − 8] = - 4

Option (a) = - (P + Q) = - 4;

8. (b) S1 = 1 + 2 + 3 + ... + n = 6 [Assuming n = 3)

S2 = 1 + 3 + 5 + ... = 9 (Assuming n = 3)

S3 = 1 + 4 + 7 + ... = 12 (Assuming n = 3)

(S1 + S3)/S2 = 18/9 = 2

9. (a) $2^{4n} - 1$ is divisible by:

Let n = 1, Number = 16 − 1 = 15 (15 = 3 * 5)

Let n = 2, Number = 256 − 1 = 255 (127 = 3 * 5 * 17)

Common factors are 3*5 = 15;

10. (b) $3^n - 2n - 1$ is divisible by:

When n = 1, Expression = (3 − 2 − 1) = 0

When n = 2, Expression = (9 − 4 − 1) = 4

When n = 3, Expression = (27 − 6 − 1) = 20

HCF of 4 and 20 is 4;

11. (a) Series is: 1, 3^1, 3^2, 3^3,....

A = 1, R = 3; Sn = 1 $[3^n - 1]/(3 - 1) > 7000$

$3^n > 14001$

$3^9 = 19683$, which is just greater than 14001

The least value of n is 9.

12. (b) Let n = 3. GP terms are 1, 2, 4

S = Sum = 1+2+4 = 7

P = Product = 1.2.4 = 8

R = Sum of reciprocals = 1 + ½ + ¼ = 7/4

Now, $S^3.R^{-3} = 7^3.4^3/7^3 = 4^3 = 64 = 8^2 = P^2$

P is GM between S^n and R^{-n}

13. (a) $X = 1/(1 - a)$; $X - aX = 1$; $a = (X - 1)/X$

$Y = 1/(1 - b)$; $Y - bY = 1$; $b = (Y - 1)/Y$

Required Sum $= 1/(1 - ab) = XY/(XY - XY + X + Y - 1) = XY/(X + Y - 1)$;

14. (a) A, B, C are in GP. Let A = 1, B = 2, C = 4

$A(B^2 + C^2) - C(A^2 + B^2) = 1(4 + 16) - 4(1 + 4) = 20 - 20 = 0$;

15. (a) Let A = 2, B = 18, X = 10, Y = 6, Z = (2*2*18)/(2+18) = 3.6

X, Y, Z = 10, 6, 3.6 are in GP

And X > Y > Z. (Equality shall not hold true)

16. (a) $A = B/3 = C/5 = K$

$A = K$, $B = 3K$, $C = 5K$

A, (B − A), (C − A) are in GP; K, 2K, 4K are in GP and that's true

A, B, C = K, 3K, 5K are in AP.

17. (c) $S1 = 1/(1 - ½) = 2$

$S2 = 2/(1 - 1/3) = 3$

$Sn = (n + 1)$

$S1 + S2 + \ldots Sn = 2 + 3 + \ldots + (n + 1) = n(n+1)/2$;

SELF ASSESSMENT TEST 10
ARITHMETIC PROGRESSION

18 Question, 18 Marks

1. The sum of three numbers in A.P. is 33 and their product is 1155. Find the second term of the series.

   a) 5 b) 7 c) 9 d) 11

2. The sum of 8th and 18th terms of an A.P. is 144. Find the sum of the first 25 terms.

   a) 1000 b) 1500 c) 1800 d) 2500

3. The sum of 16th and 26th terms of an A.P. is 200 and that of 18th and 28th terms is 600. Find the 22nd term.

   a) 50 b) 200 c) 125 d) 175

4. If three prime numbers in AP are such that that their sum is 39, then the smallest of the prime number is:

   a) 3 b) 7 c) 13 d) Data Insufficient

5. The sum of three numbers in AP is 24 and their product is 440. Find the second term of the series.

   a) 5 b) 8 c) 11 d) 16

6. The sum of 4th and 10th term of an AP is 42. Find the sum of the first 13 terms.

   a) 253 b) 263 c) 273 d) 293

7. The sum of 3rd and 5th term of an AP is 2 and that of 4th and 8th term is 10. Find the 2nd term.

   a) 2 b) − 3 c) − 5 d) None of the above

8. Find the sum of all the numbers between 100 and 200 which are divisible by 7.

   a) 2114 b) 2107 c) 2100 d) None of the above

9. What is the maximum sum of the AP: 15, 14.5, 14,...

   a) 217.5 b) 218 c) 232.5 d) 233

10. The interior angles of a polygon are in AP. The smallest angle is 120° and the common differ-ence is 5°. Find the number of sides of the polygon.

a) 9 b) 16 c) Can't be determined d) None of the above

11. The sum of 4th and 7th term of an AP is 56 and that of 5th and 8th term is 68. Find the 6th term.

a) 25 b) 28 c) 31 d) 37

12. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of the sides than contain 8 balls less than each side of the triangle did. Determine the initial number of balls.

a) 1540 b) 1210 c) 2878 d) 2209

13. The ratio between the sum of n terms of two arithmetic progressions is (7n + 1) : (4n + 27). The ratio of their 11th term is:

a) 124 : 105 b) 136 : 117 c) 148 : 111 d) None of the above

14. Find the sum of the following series till n terms: $1+5+12+22+35+.....+t_n$.

a) $\frac{1}{2}(n+1)$ b) $\frac{n}{2}(n+1)$ c) $\frac{n^2}{2}(n+1)$ d) $\frac{n^3}{2}(n+1)$

15. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. What is the common difference of the A.P.?

a) 1.5 b) 2.5 c) 3.5 d) 5.5

16. The first and the last terms of an A.P. are A and L respectively. The sum of nth term from the beginning and nth term from the end is:

a) A + 2L b) A + 3L c) A + L d) 2A + L

17. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

a) 1, 7, 13 b) 7, 13, 19 c) 1, 5, 9 d) None of the above

18. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. What is the sum of the first term, the common difference and the sum of first 20 terms?

a) 740 b) 742 c) 741 d) 743

EXPLANATORY ANSWERS

1. Sum = 33 = 3A; A = 11
   Numbers are: (11 – D), 11, (11 + D)
   Option D

2. A + 7D + A + 17A = 144
   2A + 24D = 144
   S(25) = 25/2[2A + 24D] = 25/2(144) = 1800
   Option C

3. A + 15D + A + 25D = 200; 2A + 40D = 200; A + 20D = 100
   A + 17D + A + 27D = 600; 2A + 44D = 600; A + 22D = 300
   D = 100, A = - 1900
   T22 = A + 21D = - 1900 + 2100 = 200
   Option B

4. 3A = 39, A = 13
   Numbers are (13 – D), 13, (13 + D)
   Two seta are possible: (3, 13, 23) and (7, 13, 19)
   Option D

5. 3A = 24, A = 8
   Numbers are (8 – D), 8, (8 + D)
   Second term of the series is 8. Option B

6. A + 3D + A + 9D = 42; 2A + 12D = 42
   S(13) = 13/2[2A + 12D] = 13/2(42) = 273
   Option C

7. A + 2D + A + 4D = 2; 2A + 6D = 2; A + 3D = 1
   A + 3D + A + 7D = 10; 2A + 10D = 10; A + 5D = 5
   D = 2, A = - 5
   T(2) = A + D = - 5 + 2 = - 3
   Option B

8. A = 105, L = 196, D = 7
N = (196 − 105)/7 + 1 = 14
S(14) = 14/2[105 + 196] = 2107
Option B

9. A = 15, D = - 0.5, L = 0.5
N = (0.5 − 15)/-0.5 + 1 = 30
S(30) = 30/2 [15 + 0.5] = 232.5
S(31) = 31/2 [15 + 0] = 232.5
Option C

10. Sum = (2N − 4)*90 = N/2[240 + (N − 1)5]; N = 9
Sum of interior angles of N sided polygon is (2N − 4)*90°
Option A

11. A + 3D + A + 6D = 56; 2A + 9D = 56
A + 4D + A + 7D = 68; 2A + 11D = 68
D = 6, A = 1
T(6) = A + 5D = 1 + 30 = 31
Option C

12. Total number of balls inside the triangle = 1 + 2 + 3 +... + N = N(N + 1)/2
Number of balls in each side of the square = (N − 8)
Thus, $(N - 8)^2 = N(N + 1)/2 + 669$; N = 55
Initial number of balls = 55*56/2 = 1540
Option A

13. $N/2[2A_1 + (N - 1)D_1] / N/2[2A_2 + (N - 1)D_2] = [A_1 + (N - 1)D_1/2] / [A_2 + (N - 1)D_2/2]$
Putting (N − 1)/2 = 10; we get N = 21
If we put N = 21, we get, [A1 + 10D1] / [A2 + 10D2] which becomes the ratio of their 11th term.
Ratio = (7*21 + 1) : (4*21 + 27) = 148 : 111
Option C

14. $T1 + T2 + T3 = 1 + 5 + 12 = 18$

Put n = 3, in the options.

a) $\frac{1}{2}(n + 1) = 2$

b) $n/2(n + 1) = 6$

c) $n^2/2(n+1) = 18$ – Option C

15. $A + 3D + A + 7D = 24$; $2A + 10D = 24$

$A + 5D + A + 9D = 34$; $2A + 14D = 34$

$4D = 10$; $D = 2.5$

Option B

16. Nth term from the beginning is the last term = L

Nth term from the end is the first term = A

Thus, Sum = A + L

Option C

17. $3A = 21$; $A = 7$

Now if numbers are 1, 7, 13 (as given in option A), we get:

Product of first and third term = 1 * 13 = 13, which is 6 more than the second term, i.e. 6

The three terms are either 1, 7, 13 or 13, 7, 1

Option A

18. $A + 2D = 7$

$A + 6D = 3*7 + 2 = 23$

$4D = 16$; $D = 4$; $A = -1$; $S(20) = 20/2[-2 + 19*4] = 740$

$A + D + S(20) = -1 + 4 + 740 = 743$

Option D

SELF ASSESSMENT TEST 11
GEOMETRIC PROGRESSION

17 Question, 17 Marks

1. The 9th term of a G.P. is 27 times the 6th term. What is the first term of the G.P. if the 4^{th} term is 27?

   a) 1 b) 2 c) 3 d) 4

2. The third term of a G.P. is the square of its first term. If the second term is 8, determine the 6^{th} term.

   a) 32 b) 128 c) 64 d) 1024

3. In a G.P., the ratio of the second and the fourth terms is 1 : 4 and the sum of the first and the fourth terms is 108. What is the value of the third term?

   a) 42 b) 44 c) 48 d) 52

4. If (x + 9), (x − 6) and 4 are in G.P., then find the value of x.

   a) − 16 b) − 4 c) 4 d) 16

5. How many terms of the GP 3, 3/2, 3/4 ... are needed to give the sum 3069/512?

   a) 9 b) 10 c) 11 d) 12

6. How many terms of the GP $\sqrt{3}$, 3, $3\sqrt{3}$, ... add up to $39 + 13\sqrt{3}$?

   a) 3 b) 4 c) 6 d) 5

7. The third term of a GP is the square of its 1st term. If the 2nd term is 27, determine the 16^{th} term.

   a) 3^{15} b) 3^{17} c) 3^{22} d) None of the above

8. If A, B, C are real and 5, A, B, C, 405 are in GP, find A.

   a) 45 b) 135 c) ± 15 d) None of the above

9. Four numbers form a GP in which the product of the extreme terms is 256 and the sum of the middle terms is 40. Find the sum of the four terms of the series.

   a) 32 b) 170 c) 160 d) 180

10. The second, the first and the third term of an AP whose common difference is non zero, form a GP in that order. Find its common ratio.

a) 2 b) – 2 c) 1 d) – 1

11. The number of bacteria in certain culture doubles every hour. If there were 25 bacteria present originally, how many bacteria will be present at the end of 6^{th} hour?

a) 800 b) 1600 c) $25(2)^6$ d) $2(25)^6$

12. Mr. Shyam Das is entitled to receive an annual payment from his employer, which for each year is less by 1/10th of what it was for the previous year. If the first payment is Rs. 10,000; what is the maximum amount he can receive, however long he may live?

a) Rs. 80,000 b) Rs. 100,000 c) Rs. 90,000 d) Rs. 110,500

13. If $x = 1 + a + a^2 + a^3 + a^4 + ...\infty$ and $y = 1 + b + b^2 + b^3 + b^4 + ...\infty$, then what is the value of ?

$1 + ab + a^2b^2 + a^3b^3 + ...\infty$?

a) $\frac{xy}{x+y-1}$ b) $\frac{x}{y(x+y)}$ c) $\frac{xy}{(x+y+1)}$ d) None of the above

14. After striking a floor a certain ball rebounds 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 600 metres.

a) 3600 m b) 5400 m c) 7200 m d) None of the above

15. If a, b, c, d are in Geometric Progression then $(a^2 + b^2)$, $(b^2 + c^2)$, $(c^2 + d^2)$ are in:

a) Geometric Progression b) Arithmetic Progression
c) Both a) and b) above d) None of the above

16. If a, b, c, d are in Geometric Progression, then $\frac{1}{a+b}, \frac{1}{b+c}, \frac{1}{c+d}$ are in:

a) Harmonic Progression b) Arithmetic Progression
c) Geometric Progression d) All of the above

17. If p, q, r are in AP, q, r, s are in GP and r, s, t are in HP, then p, r, t are in:

a) Arithmetic Progression b) Geometric Progression
c) Harmonic Progression d) None of the above

EXPLANATORY ANSWERS

1. $AR^8 = 27.AR^5$; $AR^3 = 27$
   $R^3 = 27$; Thus A = 1
   Option A

2. $AR^2 = A^2$; $R^2 = A$
   AR = 8; $R^3 = 8$; R = 2; A = 4
   $T6 = AR^5 = 4*2^5 = 128$
   Option B

3. $AR/AR^3 = ¼$; $R^2 = 4$; R = 2
   $A + AR^3 = 108$
   $A (1 + 8) = 108$; A = 12
   $T3 = AR^2 = 12*2^2 = 48$
   Option C

4. $(x - 6)^2 = 4(x + 9)$; $x^2 - 16x = 0$; x = 16; Option D

5. A = 3, R = ½
   $3069/512 = 3 [1 - (1/2)^n]/(1 - ½)$
   $1023/512 = (2^n - 1)/2^{n-1}$
   Putting n = 10, we get the required answer. Option B

6. A = √3, R = √3
   $39 + 13\sqrt{3} = \sqrt{3} [(\sqrt{3})^n - 1]/(\sqrt{3} - 1)$
   $78 + 26\sqrt{3} = \sqrt{3} [(\sqrt{3})^n - 1](\sqrt{3} + 1)$
   Putting n = 6, in RHS, we get = $26\sqrt{3}(\sqrt{3} + 1) = 78 + 26\sqrt{3}$
   Option C

7. $AR^2 = A^2$, $R^2 = A$
   AR = 27; $R^3 = 27$; R = 3; A = 9
   $T(16) = AR^{15} = 9.3^{15} = 3^{17}$
   Option B

8. $B^2 = 405*5 = 2025$; $B = \pm 45$

$A^2 = 45 * 5 = 225$, $A = \pm 15$

Option C

9. $A.AR^3 = A^2R^3 = 256 = 2^2.4^3$

$AR + AR^2 = 40$; $AR(1 + R) = 40$, assumption fits in

$S(4) = A(1 + R + R^2 + R^3) = 2(1 + 4 + 16 + 64) = 2*85 = 170$

Option B

10. A, (A – D), (A + D) are in GP

$(A - D)^2 = A(A + D)$

$A^2 + D^2 - 2AD = A^2 + AD$

$3AD = D^2$

$D = 3A$

Terms are, A , -2A, 4A are in GP

Common ratio = -2A/A = - 2

Option B

11. $T(6) = 25(2)^5 = 800$. Option A

12. Series is: 10000, 9000, 8100, ...

A = 10000, R = 9/10, S = 10000/(1 – 9/10) = 100,000

Option B

13. $X = 1/(1 - a)$; $X - aX = 1$; $a = (X - 1)/X$

$Y = 1/(1 - b)$; $Y - bY = 1$; $b = (Y - 1)/Y$

Required Sum = $1/(1 - ab) = XY/(XY - XY + X + Y - 1) = XY/(X + Y - 1)$

Option A

14. Distance covered = $H(1 + R)/(1 - R) = 600\ (1+4/5)/(1-4/5) = 600*9 = 5400$.

Option B

15. A = 1, B = 2, C = 4, D = 8

$(A^2 + B^2) = 5$

$(B^2 + C^2) = 20$

$(C^2 + D^2) = 80$

5, 20, 80 are in GP

Option A

16. A = 1, B = 2, C = 4, D = 8

1/(A+B) = 1/3

1(B+C) = 1/6

1/(C+D) = 1/12

1/3, (1/2*1/3), (1/4*1/3) are in GP

Option C

17. AP: 1, 2, 3

GP: 2, 3, 4.5

HP: 3, 4.5, 9

1, 3, 9 are in GP

Option B